Magnetohydrodynamic Turbulence Sustained by Alfvén Wave Reflection in the Solar Atmosphere and Solar Wind

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Abstract

Spacecraft measurements from Helios and Ulysses have clearly shown that the solar wind is a turbulent flow. The power spectra of magnetic and velocity fluctuations follow a power law scaling, a feature of well-developed turbulence, with different slopes at low and high frequencies. Moreover, the critical frequency dividing the two branches of the spectra varies with distance, indicating that turbulence is active and evolving in the expanding solar wind. In particular, for a wide range of frequencies ($10^{-4}$ Hz $< \nu < 10^{-2}$ Hz), magnetic and velocity fluctuations display a strong correlation, a property reminiscent of outward propagating Alfvén waves, which, on the other hand, cannot interact nonlinearly in order to generate the observed turbulence.

Direct and indirect observation of Alfvén waves in the inner corona and in the solar wind suggested that the above Alfvénic spectrum is the later evolution of the turbulence developed close to the solar surface. This, in turn, represents a very efficient process in transferring the energy to small scales, which places turbulence in the multitude of mechanisms that can be invoked for the heating of the corona and the acceleration of the solar wind.

Magnetohydrodynamic waves are copiously generated in the photosphere, however, the compressible modes are readily damped in the chromosphere, leaving only an outward flux of Alfvén waves as a remnant of the solar wave-like activity. The stratification induced by gravity and the birth of a fast wind in the inner corona offer a solution for the creation of a well-developed turbulent spectrum from an outward propagating flux of Alfvén waves, solving the apparent contradiction of the coexistence of Alfvénic turbulence and prevalently outgoing fluctuations, also allowing quantitative estimates of the turbulent dissipation as a mechanism for the acceleration of the solar wind. Linear WKB theory of wave propagation in a non-uniform medium shows that in order to conserve the energy flux, the wave amplitude must grow as a response of a decreasing propagation speed. These waves must be regarded as large amplitude fluctuations in the low corona, rather than linear perturbations of the mean magnetic and velocity fields. On the other hand, it is well known that nonlinear interactions among Alfvén waves occur only between counter-propagating modes, hence in absence of a downward flux, the waves coming from the Sun could in principle propagate freely in the corona and solar wind, transporting the energy in the outer heliosphere. However, gradients in the propagation
speed cause wave reflection and one ends up with two populations of waves, which can finally interact nonlinearly. Since the kinetic and magnetic Reynolds numbers are very high in the corona (low viscosity and resistivity) and the wave amplitude grows in absence of dissipation, successive nonlinear interactions lead to the development of a (incompressible) turbulent state which, in turn, allows the energy to be transferred from the large injection scales to the small ones, where dissipation mechanisms are at work. Moreover, the waves themselves do work on the wind (the stress tensor can be decomposed in a magnetic pressure, a WKB effect, and terms due to the imbalance between magnetic and kinetic energy, a non-WKB effect), contributing to the solar wind acceleration. We have studied the efficiency of such a process in the general simplified case of an isothermal wind in spherical expansion, extending previous works on the linear propagation of Alfvén waves in stationary stratified atmosphere with wind. In this semi-analytical treatment waves are stationary and a phenomenological model accounts for the nonlinear interactions and turbulent dissipation, coupling modes at different frequencies. The main results is that low frequency waves drive the non linear interaction and dissipation, as expected from the linear analysis, but also the initial spectrum at the coronal base has relevant consequences on the dissipation and on the resulting wave amplitudes. A further study applied to the solar case shows, indeed, that a Kolmogorov-like spectrum (rather than a flat spectrum) at the photosphere reproduces the observed profiles of the rms amplitudes of the fluctuations and cross helicity (a measure of the relative inward and outward dominance) in the whole corona and solar wind. However, in the chromosphere, the disagreement with observations is quite remarkable, both for the uncertainties of the data available and for the roughness of the model with respect to the nonlinear interactions and to the characterization of the atmospheric layers. Concerning the first issue we are investigating the efficiency of nonlinear interaction in a specified atmosphere (photosphere-chromosphere or corona, with wind) adopting a shell-model to follow the evolution of turbulence, while waves are no more treated as stationary and their frequency spectrum is an input of the simulation. Perpendicular wavenumber spectra, frequency spectra, energy dissipation and wave amplitudes can be derived and compared to the observations. In particular the modification of the frequency spectrum with distance can be studied, which represents a tracer for the evolution of turbulence in inhomogeneous media. The atmosphere and wind response to the wave excitations has been neglected so far. Using a (hydrodynamical) model for the wind and the previous developed semi-analytical model for the wave propagation and dissipation, one can study the properties of a turbulent solar wind driven by Alfvén wave reflection. The resulting density, speed and temperature of the wind together with the fluctuation amplitudes and cross helicity at different heliocentric distances can be compared to the observations in order to give constraints to the quantities at the coronal base, for which data are missing or not clearly interpretable.
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Chapter 1

The Sun

1.1 Introduction

The Sun and the solar wind represent a unique natural laboratory to study and test plasma physics and related processes occurring in other common astrophysical environments which, unfortunately, cannot be subjected to direct investigations and detailed measurements. Despite the enormous increase in observational and theoretical models, some very basic and fundamental questions concerning the Sun’s Magnetic activity remain unsolved. Heliosysmology has proved to be a powerful tool for the investigation of the solar interior, also, numerical simulation of the internal three dimensional structure of the Sun are becoming feasible. However, the generation and maintenance of the magnetic dynamo, which regulates the solar cycle in the convective region, is still unclear. Again, the magnetic field complexity and dynamics are the major sources of uncertainty and problems one has to face when the target of the investigation falls into the surface activity of the Sun. Particle acceleration, the heating of the solar corona, the acceleration of the solar wind are all different aspects of the same central problem, the physical processes occurring at the surface level in response to the (almost unknown) solicitations coming form the interior, were they either mechanical (direct forces) or energetical (energy deposition and flux). The magnetic field topology is still poorly known at the surface and very different plasma regimes succeed one another in a relatively small environment. These conditions allow a multitude of plasma processes and instabilities to occur, the understanding of the major driving which is behind a solar activity manifestation, if one exists, requires both theoretical and observational efforts. Furthermore, this is still an incomplete picture, since, whatever is the physical mechanism that makes these processes possible, their back reaction must be consider in studying the global dynamics of Sun interior or the interaction of the Sun with the solar system and the heliosphere. Hence, apart form the magnetic field complexity, a serious limitation comes from the fact that many processes acting in the solar atmosphere involve scales spanning
over a wide range, both in the spatial and temporal domain, allowing theoretical and numerical investigation to be feasible only in simplified frameworks. Two longstanding examples, that only recently have become to be considered as a unique process, are the heating of the corona and the acceleration of the solar wind, to which this theses is dedicated.

Studying the solar environment one can also address some important issues concerning more general aspects of astrophysics, such as the development, sustainment and evolution of turbulence. In fact, the observations available for the solar wind give us an almost complete background, in which theory can be tested. Helios, Ulysses, SOHO and ground observations, provided data on the distribution function of particles, the large scale fluctuations of the magnetic, velocity and density fields, together with the overall structure (density, speed, temperature, mass flux) of the solar wind. It turns out that large amplitudes, low frequency, Alfvénic fluctuations are ubiquitous in the solar corona, from the Sun surface to the entire heliosphere, and that turbulence can be considered as incompressible in most of the cases (except the first layers of the solar atmosphere). The long-living turbulence is advected by the wind expansion and must be sustained in some way in its path to the heliosphere. In the outer heliosphere (beyond 1 AU) pick-up ions and shear interaction between the fast and slow streams of the solar wind have been proved to be viable sources for the sustainment of turbulence. In the inner heliosphere, for which data availability is limited, the issue is more controversial since the wind is under the direct influence of the Sun activity and compressible effects, structures formation, magnetohydrodynamic (MHD) waves (and their coupling) are all important ingredients that must be considered.

1.2 The Solar Atmosphere

The solar atmosphere is usually divided in three stratified layers, the photosphere, the chromosphere and the corona. The last two regions are separated by a very thin and irregular layer, the transition region. Density decreases rapidly with height above the solar surface but temperature, after decreasing form about 6600 K (at the bottom of the photosphere) to the minimum value of about 4300 K (at the top of the photosphere), slowly increases in the chromosphere and then dramatically rises to a few million degrees through the transition region (see. fig. 1.1). Further out the temperature falls slowly in the outer corona, which is expanding in the solar wind, to a value of about $10^5$ K at the Earth orbit. The reason for the temperature rise above the photosphere has been one of the major problem in solar physics and has not been solved yet. The chromosphere is probably heated by sound waves generated in the convection zone which propagate outward, steepen and hence damp their energy through shock formation; higher levels may be heated by several magnetic mechanisms, involving magnetohydrodynamic wave coupling and dissipation,
current sheet formation and in general magnetic field dissipation.

The Photosphere  The photosphere is a very active region, being under the continuous influence of the turbulent convective motions developed in the underlying convective region and is usually seen in white light since it emits at all wavelengths with some absorption lines given by the overlying atmosphere. Its activity is observed as small granular motion resulting from the emergence of convective cells, the so called granulation, which covers the whole Sun at the photospheric level. The center of the granule is brighter and made up of hot rising (0.4 km s\(^{-1}\)) and horizontally (0.2 km s\(^{-1}\)) outflowing plasma while at the cells' edge the material is downflowing. Their typical size is 1000 km and their duration of a few minutes. In addition to the small scale granular velocities other motions includes large scale velocity patterns, known as mesogranulation and supergranulation, depending on their size, 5000 km and 34000 km respectively. Supergranule cells are irregular in shape and can last up to 1-2 days, they show very little brightness variation in the photosphere and are best recognized at the limb as a horizontal velocity pattern. Material is rising at the center at 0.1 km s\(^{-1}\), moves horizontally towards the edges at 0.4 km s\(^{-1}\) where it descends at 0.1 – 0.2 km s\(^{-1}\). Their boundaries are very prominent in the chromosphere and are region where magnetic flux is concentrated. The magnetic field is very irregular and made up of small magnetic elements that are shuffled around and evolve rapidly, forming large scales pattern of several types. Among them, the large scale unipolar areas contain elements of a predominant polarity and extends for about 100000 km in latitude and longitude.
They last for years and rotate faster then the photospheric plasma, with less differential rotation, which may imply that they are anchored deep in the Sun. The underlying cause of unipolar regions is believed to be the slow diffusion of an active region\(^1\) or it may be associated to a giant convection cell. Above some large unipolar region one may find coronal holes, which are thought to be the locations where the fast streams of the solar wind originate. Most of the photospheric magnetic flux outside active regions is concentrated at the supergranule boundaries (network field) and is made up of several intense flux tubes. Their typical size is 100 – 300 km and their typical field strength is about 1 – 2 kG. Magnetic elements are also found within supergranule cells and known as inner network fields. Above the supergranule the magnetic field lines are thought to form a magnetic canopy (horizontal magnetic field over a field free region) at whose boundaries magnetic field lines open up in their expansion to the upper layer.

The Chromosphere  The temperature increases rapidly in the boundary between the middle and upper chromosphere and at the transition region (which are usually considered as a whole). However the height at which the rise occur may depend on the region considered on the Sun. Indeed the chromosphere is highly non-uniform, emission of the \(^\text{Ca} II \text{ K}\) line reveals the network of supergranule boundaries very clearly as a bright pattern. At the limb the chromosphere is seen as collection of plasma jets, known as spicules, ejected from the high chromospheric part of the supergranule boundaries. At the poles, where field lines are open, larger jets of chromospheric material are found, called macrospicules. Spicules have temperature of about 1 – 2 \(10^4\) K, electron density of about \(3 \times 10^{10-11}\) cm\(^{-3}\) and lifetimes of 5-10 minutes.

The Transition Region  The transition region is observed mainly in EUV emission lines. Rather then being a static horizontal layer, it is probably composed of many dynamic thin sheaths around the cool spicules which are continually intruding into the coronal plasma. Typical models of the transition region predict a the temperature rise from \(3 \times 10^4\) K to \(3 \times 10^5\) K in only 30 km, and a slower rise up to \(10^6\) K in the following 2500 km. Moving higher in the solar atmosphere the magnetic field above supergranule continues to spread out, therefore the magnetic network is seen to thicken with increasing temperature and eventually at coronal heights it disappears.

The Corona  The corona is seen in white light only at eclipses or by means of coronographs, since the photosphere is much brighter. The observed faint halo

\(^1\)Active regions are generated by a prolonged emergence of a magnetic flux region from below the photosphere. They are bipolar and their late evolution is that of two separated sunspot of opposite polarity joined by magnetic filaments which expands up in the so called coronal loops.
1.2. The Solar Atmosphere

Figure 1.2: \textit{Left panel}. The boundary of a polar coronal hole from coronograph data (solid line), X-ray photographs (dashed line) and K-coronometer data (dotted line). \textit{Right panel}. The variation of the electron density ($N_e$) with distance from the solar center in units of solar radius. Also shown is the inferred velocity $V$ (assuming a particle flux at 1 AU of $3 \times 10^{18} \text{ cm}^{-2} \text{ s}^{-1}$) and the velocity of radially flowing isothermal ($T = 2 \times 10^6 \text{ K}$) solar wind (Munro & Jackson 1977).

of very low density and high temperature is due to the scattering of the photospheric light both off electrons (K-corona) and off dust (F-corona). Within $2.3 R_\odot$ the K-corona is dominant and its intensity is proportional to the electron density, appearing brighter where more plasma is situated. The overall shape of the corona varies with the solar cycle, near sunspot maximum, bright features called \textit{streamers} extend out in all directions, near sunspot minimum streamers are present only in the equatorial region and \textit{polar plumes} are seen to fan out from the poles. Coronal streamers are roughly radial structures extending from heights of $0.5 R_\odot$ up to $1 - 10 R_\odot$ with a density enhancement of 3 to 10 (helmet streamers lie above prominences, active-region streamers above active region, see fig. 1.3). Polar plumes are ray-like structures near the poles, they are denser and cooler than the interplume plasma, last for only 15 h and presumably outline the local magnetic field (plumes are also seen in coronal holes). In \textit{X-rays} the corona emits thermally and can be viewed directly since the contribution of the lower atmosphere is negligible. Two types of region can be distinguished. Those in which the magnetic field lines are predominantly open appear relatively dark and are known as \textit{coronal holes}, here the plasma is flowing outward to produce the solar wind. Those in which the magnetic field is mainly closed consist of a multitude of \textit{coronal loops}. Also small intense feature, called X-ray bright points are scattered over the whole disc. The latter have typical diameter of $22000 \text{ km}$ and lifetime of 8 h but some can last up to 2 days. They are a coronal manifestation of the tiny bipolar areas of
emerging flux and appear to consist of several loops (most of the emerging flux shows up as bright points rather than active regions). Coronal loops have been extensively studied and classified in relation to the coronal heating problem. Interconnecting loops join different active regions and seem to form either when two loops stretch from separate active regions and reconnect or when one loop reconnects with some newly emerging flux. Their ends are rooted in islands of intense magnetic field near the edge of active regions. A single loop last about a day but a whole loop system may endure for many rotations. Their typical temperature is about $2 - 3 \times 10^6$ K and density approximately $7 \times 10^8$ cm$^{-3}$. Quiet loops do not connect active regions and in soft X-rays they are somewhat cooler $T \approx 1.5 - 2.1 \times 10^6$ K and their density varies on a wider range $2 \times 10^8 - 10^9$ cm$^{-3}$.

Coronal holes are extended regions with lower density (a factor of three) and lower temperature than the typical background corona, their geometry and the variation of density with height are shown in fig. 1.2. Coronal holes have an open diverging magnetic structure and lie above some of the more extensive large scale unipolar regions in the photosphere. In the photosphere and low chromosphere the plasma properties are indistinguishable from those of their surroundings. At transition region temperature they appear quite different: the pressure is generally two or three times smaller and the temperature gradients five time smaller than in the normal quiet Sun; this makes the transition region five times thicker and the conductive flux an order of magnitude smaller. The dominant energy loss is from convective transport by the solar wind rather than from the downward conductive flux, at the base of the hole the steady outflow is probably about $16 \text{ km s}^{-1}$. Coronal holes endure for several rotations and the longer-lasting polar coronal holes disappear only near sunspot maximum. Their boundary is occupied by an arcade of coronal loops (see fig. 1.3) and whether or not a coronal hole forms seems to depend on the width of the unipolar region. If the region is narrow, the bounding arcades are not very high and can be surmounted by a single streamer. When the unipolar region is wider than $3 - 4 \times 10^5$ km, fields lines from the center of the region reach such a height that they can be pulled out by the solar wind. In practice, the emergence of an active region may change the large scale coronal magnetic configuration and it may stimulate the formation of a coronal hole, which often joins with the polar hole of the same polarity.

1.3 The Solar Wind

In open field regions the solar corona is not in hydrostatic equilibrium but it is continuously expanding outward as the solar wind. Most of it escapes along open field lines along coronal holes, especially the two polar coronal holes, but small open regions above active regions also contribute. The two polar holes, with the magnetic field oppositely directed, are separated by the closed magnetic configuration typical of active regions at small distance from
the surface, but beyond a few solar radii they come into contact at a neutral (heliomorphic) current sheet. The neutral current sheet is neither regular as a disc nor lying on the equatorial plane, it is warped because of the presence of the large scale photospheric field and also inclined by about 7° to the Earth’s orbit. As the Sun rotates, an observer above the solar equator sees a sequence of alternating polarities, successively from one side or the other of the current sheet (see left panel in fig. 1.4). Near sunspot maximum the current sheet is highly distorted but near sunspot minimum the warping is slight. Close to the solar surface the magnetic field dominates the plasma dynamics which is mainly corotating with the Sun. At a certain distance (the Alfvén radius) the situation is reversed and the magnetic field is carried by the plasma, the rotation of the Sun makes the field to assume a spiral configuration while the plasma flows roughly radially (see right panel in fig. 1.4). Ulysses, with its near polar orbit from 1.4 to 5.4 AU, has given us a graphic picture (fig. 1.5) that solar wind comes in two states: an irregular slow wind with typical speeds of $400 \text{ km s}^{-1}$ and a smooth fast wind with a speed of $750 \text{ km s}^{-1}$. At times of lower solar activity (left picture), the solar wind is bimodal, consisting of a dominant quasi-steady high-speed wind that originates in open-field polar coronal holes, filling much of the heliosphere, and a variable, low-speed wind that originates around the equatorial streamer belt. With increasing activity, this orderly bimodal configuration of the corona and the solar wind breaks down, as the polar holes shrink and streamers appear at higher and higher heliographic latitudes. At these times, the bimodal wind structure is replaced by a complex mixture of fast flows from smaller coronal holes and transients, embedded in a slow-to-moderate speed wind from all latitudes (right picture).
Figure 1.4: Left panel. A schematic drawing of the warped current neutral sheet that was present during the summer of 1973, when Skylab observed a large coronal hole stretching across the equator. The neutral sheet probably crosses the solar equatorial plane four times (Hundhausen 1981). Right panel. Interplanetary magnetic field lines (in the solar equatorial plane) resulting from the extension of the solar magnetic field by a 300 km s$^{-1}$ (constant speed) solar wind. Magnetic stresses in the extended solar field and angular momentum imparted to the solar wind by angular velocity of the Sun are neglected (Parker 1963).

<table>
<thead>
<tr>
<th>Property @ 1 AU</th>
<th>Slow Wind</th>
<th>Fast Wind</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow speed</td>
<td>400 km s$^{-1}$ ($\approx 50%$),</td>
<td>750 km s$^{-1}$ ($\approx 5%$)</td>
</tr>
<tr>
<td>Density</td>
<td>7 cm$^{-3}$ ($&gt; 50%$)</td>
<td>3 cm$^{-3}$ ($&lt; 50%$)</td>
</tr>
<tr>
<td>Temperature</td>
<td>$T_p \approx 200000$ K ($&gt; 50%$)</td>
<td>$T_p \approx 50000$ K ($&gt; 50%$)</td>
</tr>
<tr>
<td>Composition</td>
<td>Depends on FIP</td>
<td>Independent of FIP</td>
</tr>
<tr>
<td>Freezing-in Temperature</td>
<td>$\approx 1.5 \times 10^6$ K</td>
<td>$\approx 10^6$ K</td>
</tr>
</tbody>
</table>

Table 1.1: General properties of the solar wind at 1 AU. In parenthesis is indicated the variance of the quantity (large $> 50\%$ and small $< 50\%$). FIP is the abbreviation for First Ionization Potential.

The proton temperature from the Ulysses fast latitude scan is shown in fig. 1.6. There is, yet again, a sharp boundary between fast and slow wind but the variance in the fast wind is $\approx 50\%$ rather than the $5\%$ variance in speed. This is a true variance that is difficult to reconcile with the smooth flow speed shown in the dial plot. It may be the consequence of filamentary structures in the corona such as plumes but this is still unclear. Plumes permeate all coronal holes, yet are invisible in the solar wind. How this variable, filamented flow becomes the uniform fast wind, whether this is related to the source and evolution of MHD turbulence in the solar wind is still unclear. Plumes are seen to originate in the photosphere, diverge along with the coronal hole boundaries, and extend here to at least $6 R_\odot$. Recent observations show that plumes are sometimes detectable out to $15 - 30 R_\odot$. A plume will tend to last for several
1.3. The Solar Wind

Figure 1.5: Solar wind observations (wind speed and magnetic polarity as a function of heliolatitude) collected by the Ulysses spacecraft during two separate polar orbits of the Sun, six years apart, at nearly opposite times in the solar cycle, overlaid with three concentric images taken with the NASA/GSFC EIT instrument (center), the HAO Mauna Loa coronograph (inner ring) and the NRL LASCO C2 coronograph (outer ring). Each 1-hour averaged speed measurements has been color coded to indicate the orientation of the observed interplanetary magnetic field: red for outward pointing and blue for inward. Near solar minimum (left) activity is focused at low latitudes, high speed solar wind prevails, and magnetic fields are dipolar. Near solar maximum (right), the solar winds are slower and more chaotic, with fluctuating magnetic fields.

days but within a plume there are brightness fluctuations on much shorter timescales. The slow wind has a comparable variance but with differing statistical properties and with several large spikes which may be due to the small Coronal Mass Ejections that occur even at sunspot minimum.

The proton temperature in the fast wind is also anisotropic (left panel in fig. 1.7), being larger perpendicular to the magnetic field than parallel to it and this will be seen to have a coronal counterpart in SOHO/UVCS observations. Temperature anisotropy is a diagnostic used to distinguish between suspected coronal heating processes because it tests whether high frequency Alfvén/cyclotron waves may be involved.
1. The Sun

Figure 1.6: Proton temperature as measured by Ulysses spacecraft as a function of latitude.

Figure 1.7: Left panel. Contour of solar wind proton velocity distribution functions in fast wind at 0.29 AU measured by Helios spacecraft. Contours are 0.8, 0.6, 0.4, 0.2, 0.1, 0.03, 0.01, 0.003 and 0.001 of the maximum phase space density. The distribution is anisotropic ($T_\perp > T_\parallel$), hot, and has a faster component along the magnetic field direction (dashed line) (Marsch et al. 1982). Right panel. Helium ion speed, O and C coronal freezing-in temperature, and Mg/O and Fe/O abundance ratio. The Ulysses/SWICS data are repeated to facilitate recognition of the sharp boundary between fast and slow wind (Geiss & Witte 1996).

Fast wind is relatively steady and also relatively simple in composition. In the right panel of fig. 1.7 the wind speed and the freezing-in temperature $^2$ is shown for Oxygen and Carbon in the top panel. The charge state distribution is

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$^2$At a certain height the expansion of the solar wind stops collision to be effective and the temperature of a given element is given by the temperature at such a height (freezing-in temperature)
characterized by a single, low freezing-in coronal temperature for each element of about $\approx 1 \times 10^6$ K. The composition is least biased in the fast wind (closely resembling photospheric composition) as shown by the abundance of Mg and Fe relative to Oxygen in the bottom panel of the same figure. Conversely, Mg and Fe are overabundant and the freezing-in temperatures are high and variable in slow wind. These close correlations with flow speed for a coronal process (freezing-in temperature) and a chromospheric process (composition) show that the boundary between fast and slow wind is a sharp boundary extending all the way down to the chromosphere. Fast wind is permeated by an evolving field of magnetohydrodynamic (MHD) turbulence which is presumed to be a remnant or imprint of the coronal acceleration process. Slow wind is highly variable in speed (fig. 1.5) and more complicated than fast wind in its other characteristics (fig. 1.7). The charge state distribution can no longer be characterized by a single freezing in temperature. MHD turbulence in slow wind is less evolved and more intermittent than in fast wind.

SOHO and interplanetary scintillation results show that fast wind reaches its terminal speed by $10 R_\odot$ and that at $4 R_\odot$ it is already being accelerated. At $4 R_\odot$ the temperature of heavy ions is much larger than that of protons while the difference is smaller at 1 AU. The proton temperature at $4 R_\odot$ in coronal holes is two to three times higher than the electron temperature inferred from charge state measurements in the terminal wind while they differ by less than a factor of two at 1 AU. Inferred ion temperature anisotropies are enormous between 2 and $10 R_\odot$ and are believed due to an Alfvén or ion-cyclotron wave field contributing to the perpendicular temperature. A true proton temperature anisotropy exists in the 1 AU fast solar wind, but is smaller than inferred from the coronal observations.

SOHO has hinted at some remarkable features for the polar photosphere. But, since the poles cannot be viewed effectively by SOHO (or any spacecraft confined to near the ecliptic plane), these features remain poorly defined. These findings are: (1) the rotation rate at higher latitudes is 10% to 20% lower than expected; (2) there is some evidence for a polar vortex; (3) there is some evidence of a polar concentration of magnetic flux; (4) measurements of surface and subsurface motion indicate meridional flows that are a factor of more than two higher than previously estimated; (5) there are indications that small and large scale magnetic fields on the Sun are rooted at different depths in the convection zone. These results, combined with the more general SOHO result that magnetic flux is replaced very rapidly everywhere on the surface of the Sun (approximately every 40 hours) suggest the importance of a close examination of the photospheric dynamics and magnetic field to extend our understanding of how these relate to the flow of energy into the corona.
1.4 The Coronal Heating and Acceleration Problem in Open Field Regions

The existence of the corona requires an input of energy above that which would occur by thermodynamic relaxation from the photospheric input. This input can be supplied either by thermal conduction from the hot corona or that part of MHD wave flux (responsible for heating the corona) that does not supply its energy directly to the thermal conduction flux. For a typical high speed stream energy flux density of \(1.1 \text{ erg cm}^{-2} \text{ s}^{-1}\), it follows that the outward energy flux density at the base of the coronal hole has a value of about \(7 \times 10^5 \text{ erg cm}^{-2} \text{ s}^{-1}\) (assuming an areal divergence a factor 7 greater than an \(r^2\) divergence, Munro & Jackson 1977). This value is much larger than that corresponding to thermal conduction from the corona to the chromosphere and is larger than that associated with radiation from the corona (see tab. 1.2). Thus, contrary to what expected from spherically symmetric solar wind models, it is clear that the solar winds plays a dominant role in the coronal energy balance in coronal hole regions and probably in all other open field line regions.

The excess of the non-thermal (sometimes called mechanical) energy to sustain the solar corona is just a small fraction of the total solar output. Estimates show that the total energy budget is just approximately \(10^{-4}\) of the Sun’s total energy output making it, in theory, relatively easy to suggest mechanisms to channel 0.01% of the total solar output into heating the corona. The restructuring of the solar magnetic field, modifying the corona and wind, gives important information about the general sources of the fast and slow winds, but the physical processes that accelerate the different wind speeds are still not understood. However, a strong constraint which allow to select among possible mechanisms is given by the observations from SOHO’s Ultraviolet Coronagraph Spectrometer (UVCS), which imply that the wind acceleration operates over the range of 2 to 10 \(R_\odot\) in coronal holes.

The heating ad acceleration process comprises three phases: the generation

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3 That the solar wind originates in these coronal funnels was recently found by Tu et al. (2005), who identified, by means of correlations between Doppler shifts and the coronal magnetic field as obtained by extrapolation from photospheric magnetograms, the source regions of the plasma outflow.
1.4. The Coronal Heating and Acceleration Problem in Open Field Regions

of a carrier of energy; the transport of energy into the solar atmosphere; and, fi-
nally, the dissipation of this energy in the various structures of the atmosphere. Usually it is not difficult to establish a theory that drives an energy carrier without contradicting observations. Examples of the energy carriers are the slow and fast MHD waves, Alfvén waves, magneto-acoustic gravity waves and current sheets. The heating mechanisms can be classified on the base of the timescales involved. If the characteristic timescale of the perturbations is less than the characteristic times of the back-reaction of the plasma, MHD waves are good approximations for describing energy propagation and we speak about alternating current (AC) heating mechanisms; on the other hand, if perturba-
tions have low frequencies, very narrow current sheets are formed resulting in a direct (DC) heating mechanism. In the solar atmosphere the emerging flux tubes are shaken and twisted by photospheric motions (granular motions and acoustic oscillations): if the characteristic time of these photospheric footpoint motions is much less than the Alfvénic transient time, the perturbations prop-
agate in the form of various MHD waves. This is believed to be the primary source of energy carrier for the open field regions, while for closed field region the DC scenario is equally attracting.

The major unresolved problem relating to the AC coronal heating lies in de-
termining what processes effect the conversion of MHD wave energy to flow energy. Several mechanisms have been suggested and studied till now. Slow MHD waves propagating in a stratified atmosphere steepen and form shocks, they are mainly damped in the chromosphere and weakly contribute to the coronal heating. Fast MHD waves are believed to survive until the corona and their dissipation is achieved through the Landau damping. Alfvén waves have been extensively studied since they are an exact solution of the nonlinear MHD equations and propagate undamped through the lower atmospheric layer, reaching the corona and the solar wind (in which they are observed at the Earth orbit as well). The problem with them is again that of dissipation. Nonlinear mode coupling can channel the energy in other MHD modes, achieving dissipation. Other mechanisms proposed are phase mixing, resonant absorption, parametric decay and finally turbulent heating. None of them, at the moment, seems to be able to fully account for the heating and for the acceleration required to give the wind the observed terminal speed.

The ultimate dissipation in MHD heating mechanisms involves joule heating or, to a less extent, viscosity. MHD theory is derived under the assumption of low frequencies and is not able to describe particle-wave interactions, which actually are responsible for the acceleration and the heating, and are usually handled within kinetic theories. A complete treatment of the heating and acceleration problem would require to consider a wide range of time and length scales. The MHD mechanisms mentioned above aim to describe the transport of energy from the photosphere to the corona and from the large to the small scales at which kinetic processes work.
1.5 Outline of the Thesis

In this thesis we study the efficiency of incompressible turbulence in transferring energy to small scales and in dissipating it. Turbulence is generated by the continuous nonlinear interactions among counter-propagating Alfvén waves. Generally, high frequency Alfvén waves, propagating outward along the magnetic field lines of an open flux tube, escape form the solar atmosphere in the expanding solar wind, basically without any damping. However, if the wave have low frequencies, they suffer reflection from the density gradient induced by the stratification. The wave energy is hence partially trapped in the lower atmosphere but, most important, nonlinear interactions are triggered and turbulence develops. Dissipation is modeled, at best, by means of resistive and viscous coefficients and does not take into account the kinetic effects mentioned above, while propagation is restricted to the parallel direction with respect to the mean magnetic field, assumed to be aligned with the radial gravitational field. Ultimately the efficiency of the heating mechanism studied refers to the ability of transferring energy to smaller scale at the expense of the wave energy which naturally would escape into the solar wind.

We start with a brief review of current knowledge and recent works (with related results) concerning the acceleration of the fast solar wind through wave heating mechanisms (chapter 2). It follows a resume of the properties of turbulence in the solar wind, derived from observations (mainly measurements from Helios and Ulysses spacecrafts), with emphasis on the relation between Alfvén waves and turbulence in the so called “Alfvénic” range of frequencies, commenting the efforts which have been made for its interpretation.

In chapter 3, we use a semi-analytical model to investigate the large scale properties of turbulence, modeling nonlinear interactions and turbulent dissipation with a phenomenological term. We first study the resulting dissipation varying the wave amplitudes and for different kinds of frequency coupling, considering a spherically symmetric atmosphere permeated by isothermal winds at different temperatures, and allowing for a non-spherically symmetric expansion of the magnetic flux tube. Then, the semi-analytical treatment is applied to a solar-like atmosphere, extending from the photosphere to the Earth orbit, with the inclusion of a transition region. Initially the atmosphere is specified in order to test the ability of the model in reproducing observational constraints. This assumption is then relaxed and the wave-turbulence equations are integrated simultaneously with the wind equations (continuity, momentum and energy one-dimensional equations) in order to find a stationary solution for a turbulent driven fast wind, although limited to the corona. In chapter 4, we come back to the specified atmospheric model. A coronal and photospheric layers are considered separately. Nonlinearities are treated with a 2D shell model, accounting for the turbulent dynamics which develops in the planes perpendicular to the direction of propagations, containing the fluctuating fields. The heating rate, perpendicular wavenumber spectra and frequency spectra are
obtained as a function of the radial distance for different kinds of forcing and boundary conditions, which allows to understand the efficiency of the reflection mechanism in relation to the development and sustainment of turbulence. Finally, in chapter 5, a summary and discussion of the results, together with the possible improvements and extensions of the current work, are presented.
1. The Sun
Chapter 2

The Fast Solar Wind, Heating and Acceleration Mechanisms

The discover of the solar wind, both theoretically and observationally, dates back to the end of the 50’s. The existence of a solar wind, as opposite to a static corona, was first pointed out by Parker (1958) and in 1959 the first direct observations and measurements of the strength of the solar wind were made by the soviet satellite Luna 1. In Parker’s original model gravity was overcome by the strong pressure gradients in the very hot corona and terminal speed of about 1000 km s$^{-1}$ were produced with a mean coronal temperature of about 3–4 $10^{6}$ K. In the 70’s and 80’s it became evident that even the most sophisticated solar wind models could not produce a fast wind without deposition of heat or momentum in some form in the corona, because of the smaller temperature actually inferred from observations in coronal holes. Ground observations and space missions provided a wealth of observational constraints to which models are continuously addressed for their validation or for their construction. Up to now a certain consensus has grown around a “standard model” of the fast solar wind (Hansteen et al. 1999), characterized by high ion temperatures in the corona, low electron density and temperature in the inner corona, a terminal flow speed $u_{\infty} \approx 700$ km s$^{-1}$ and a proton flux at 1 AU, $(n_{p}u_{p})_{E} \approx 2 \times 10^{8}$ cm$^{-2}$ s$^{-1}$. The characteristic of these models is that the heating and acceleration of the wind is accomplished through some process that deposits energy (heat) at around 2 $R_{\odot}$. As already stressed, the present key-problem is that of finding which is the physical mechanisms able to reproduce the source of the extended coronal heating as required by both energetics-based theoretical considerations and observational data. The latter may be summarized in the following short list (Cranmer 2004):

- Wind models driven by pressure gradients cannot be consistent with the measured terminal speed at 1 AU and the low temperature in coronal holes (especially electron temperature, $T_{e} \lesssim 1.5 \times 10^{6}$ K) without invoking some kind of additional energy deposition.
• The decrease of proton and electron temperatures with heliocentric distance deviates substantially from those derived assuming an adiabatic expansion law. The measured gradients are shallower indicating a gradual energy addition (e.g., Phillips et al. 1995; Richardson et al. 1995).

• SOHO has provided more direct evidence for extended heating. Observations from the Ultraviolet Coronograph Spectrometer (UVCS) have shown that heavy ions are both hotter and faster than protons. Strong anisotropies in the perpendicular and parallel temperature (relative to the mean magnetic field) exist for protons and ions with $T_\perp > T_\parallel$ (Kohl et al. 1997; Noci et al. 1997; Li et al. 1998; Giordano et al. 2000). Moreover, the Solar Ultraviolet Measurements of Emitted Radiation (SUMER) has suggested that the preferential heating of ions may begin very close to the solar surface, where collisions are thought to be still important (e.g., Tü et al. 1998; Peter & Vocks 2003; Moran 2003).

### 2.1 Energetics of the Fast Solar Wind

Independently of the specific mechanism, energy is deposited in the coronal gas and let this unknown source be the “mechanical” flux. If the heat is supplied to the low density plasma of the corona, the temperature rises until the energy losses balance the heating rate. These are the radiative losses, thermal conduction and acceleration of the solar wind. Consider the problem as restricted to a flow tube in which the energy flux propagates along this tube and assume also that the conductive flux is along the tube. In this conditions, and in a steady solar wind, the energy flux passing through any cross section of the flow tube is constant and can be expressed as:

$$F_0 = F_m + F_q + F_{rad} + F_{sw},$$

(2.1)

where $F_m$ is the mechanical energy flux, $F_{rad}$ the integrated radiative losses, $F_q$ the conductive flux and $F_{sw}$ is the energy flux in the solar wind. Now, let’s evaluate the various terms in the equation, locating the base of the representative solar wind model at the top of the chromosphere $r = r_0 = 1 R_\odot$. Here $F_{rad,0}$ is set to zero (no downward net radiative losses), the high density in chromosphere makes radiation efficient in getting rid of the heat conductive flux coming from the corona, so that also $F_{q,0} = 0$. The temperature and bulk speed of the solar wind are small at the top of the chromosphere, hence the solar wind energy flux is given by the potential energy $F_{sw} = (-\dot{M}) \times (-1/2v_g^2)$, where $\dot{M}$ is the mass flux and $v_g \approx 618$ km s$^{-1}$ is the escape speed at the surface. Equation 2.1 gives:

$$F_0 = F_{m,0} + \dot{M} \frac{1}{2} v_g^2,$$

(2.2)

where $F_{m,0}$ is the energy flux from the Sun. Evaluating the same relation far from the Sun, one may neglect the heat conduction as well as the solar wind
potential energy and enthalpy fluxes. Assuming that the mechanical energy acts only close to the base it follows:

$$F_0 = F_{\text{rad,}\infty} - M_{\infty} \frac{1}{2} u_{\infty}^2,$$

where $u_{\infty}$ is the asymptotic wind speed. Combining the two expressions, the energy balance may be written as

$$F_{m,0} = -\dot{M} \left( \frac{1}{2} v_g^2 + \frac{1}{2} u_{\infty}^2 \right) + F_{\text{rad,}\infty},$$

The mechanical energy flux deposited in the corona can be spent to accelerate the solar wind or can go towards radiation form the corona or towards inward conduction and radiation form the transition region. If the energy deposition by the mechanical energy occurs a coronal density scale height ($\approx 0.1 R_\odot$) above the solar surface, the energy losses from the wind flow are larger than radiative losses and the other inward fluxes (see tab. 1.1 in sec. 1.3). In fact, independently of the extent and exact location of the energy input, the energy losses through radiation which occur above the top of the chromosphere account for less than 15% of the total energy flux which is dissipated in the corona. The energy balance in the open corona is hence well approximated by the first term on the RHS of eq. 2.4 that can be recast in an expression for the mass flux,

$$-\dot{M} \approx \frac{F_{m,0}}{\frac{1}{2} v_g^2 + \frac{1}{2} u_{\infty}^2}.$$

Recalling that the escape speed and the terminal speed are of the same order of magnitude it turns out that the mass flux is proportional to the amount of mechanical energy flux dissipated in the corona and the relation above can be used to roughly evaluate the energy input in solar wind models. Assuming a divergence of the flux tube geometry greater than radial by a factor $f_{\text{max}} \approx 7$ (Munro & Jackson 1977), the energy flux density consistent with the solar wind energy mass loss, $\dot{M} = 2 \times 10^{-10}$ g cm$^{-2}$ s$^{-1}$, is given by $f_{m,0} / A_0 \times f_{\text{max}} \approx 10^5 \times f_{\text{max}}$ erg s$^{-1}$ cm$^{-2}$, with $A_0$ the cross section at $r = r_0$.

Nonetheless, the small amount of radiation flux has important consequences on the properties of chromospheric plasma because it regulates the pressure of the transition region influencing the mass flux of the wind. If, in fact, the mechanical flux is increased the energy transported downwards into the transition region is also increased. As a consequence the density goes up in order to have sufficient matter to radiate away the excess of energy. This affects also the coronal density and the solar wind mass flux increases. In multifluid, standard models most of the heating is supplied to ions, most of the energy flux remains in the ions and electrons play a minor role in accelerating the solar wind. An alternative to such standard models is obtained when most of the heating goes into electrons rather than ions. However to attain a terminal wind speed and a a mass flux compatible to observational constraints
such models require large electron temperatures in the inner corona (about $3 \times 10^6$ K) which is much higher than the inferred freezing-in temperature (below $10^6$ K). This disagreement, however, does not rule out completely these models since the classical heat conduction, derived under the assumption of a collision dominated plasma, may not be valid in corona or perhaps at the transition region. This is also relevant for standard models since the electron heat flux is crucial for building and maintaining the transition region.

\section*{2.2 Heating Mechanisms}

As discussed above, in open field regions the corona loses energy mainly by the solar wind convective flux. Such energy losses must be balanced by an adequate flux from the photosphere. Estimations form coronal models lead to a value $\approx 5 - 8 \times 10^5$ erg cm$^{-2}$ s$^{-1}$ (Withbroe 1988). It is quite natural to think that the energy associated to photospheric motions can be transported in the corona through the magnetic field and there dissipated. This energy reservoir can be estimated computing the pointing flux $S = \left(\frac{c}{4\pi}\right)E \times B$, where $c$ is the speed of light, $B$ is the magnetic field and $E$ is the electric field induced by the plasma motions. The latter can be estimated as $E = -\delta v \times B_0/c \approx 10^7$ erg cm$^{-2}$ s$^{-1}$ assuming a negligible resistivity and using the value $B_0 \approx 100$ G for the background magnetic field and $\delta v \approx 0.5$ km s$^{-1}$ for the velocity fluctuation: hence this mechanical flux is sufficient to balance the coronal energy losses. Wave based mechanisms (AC processes according to sec. 1.4) have to explain how energy is transported to the corona and how it is dissipated. The main difficulty concerning the first point is given by the transmission properties of the chromosphere and transition region for the MHD waves produced by the photospheric motions. Retaining the usual classification in slow, fast and Alfvén waves (though strictly valid only in homogenous media) to distinguish the various MHD modes, the following considerations are commonly accepted. Observations show that the flux associated to slow magneto-acoustic waves is insufficient to heat the corona unless the wave amplitude is larger than the observed non-thermal velocity. Fast magneto-acoustic waves suffer strong reflection from steep gradients and are not able to reach the corona. The remaining Alfvén mode, to which fast mode reduces in the limit of parallel propagation, is believed to be the most plausible candidate for transferring energy to the corona, since, even if Alfvén waves are reflected at the transition region, a sufficient amount of photospheric flux (about 20\%) is transmitted to the corona.

Let's hence consider dissipation. We will not examine the whole set of AC heating mechanisms known in literature, since it would require an entire chapter or more. We give just some examples of mode coupling, which by itself is not a proper heating mechanism, to show how rich is the subject and to give a flavor of the difficulty one encounters in identifying a dominant mechanism responsible for the heating (if one exists).
2.2 Heating Mechanisms

The basic problem with heating is that the classical collisional dissipation is very small, Reynolds numbers in the corona are of the order of $10^{11}$ and the only way to dissipate energy is to create small scales, at which some (or unknown) process is able to accelerate and heat the plasma particles. This small scale generation can be produced by the interaction of waves with the inhomogeneous background medium or by nonlinear interactions among waves which lead to an energy cascade to small scale structures. The latter mechanism may be imputed to the development of MHD turbulence while examples of the former are resonant absorption and phase mixing. Turbulence will be considered in the next section 2.3, since this wide topic involves heating not only in the acceleration region of the corona but also in the more extended heliosphere and addresses general aspect of turbulent dynamics, not necessarily restricted to the solar wind acceleration problem.

Both resonant absorption and phase mixing are triggered by the inhomogeneity of the equilibrium field in the direction perpendicular to the mean magnetic field, and involve waves travelling along the mean magnetic field but with different polarizations. Consider an equilibrium magnetic field pointing in the $z$ direction and varying along the $x$ coordinate from $B_1$ to $B_2$, as in Fig. 2.1, and a magneto-acoustic surface wave travelling along the field line with polarization in the $x$ direction (resonant absorption). Its phase speed is given by $v_{ph} = \sqrt{(B_1^2 + B_2^2)/(4\pi\rho_1 + \rho_2)}$ so that at some position $x_0$ it becomes equal to the local Alfvén speed $V_{A0} = B(x_0)/\sqrt{4\pi\rho(x_0)}$. Consider the evolution of the wavefronts of the surface wave in such a configuration (solid and dashed line for the peak and the trough, respectively, in panel $a$). Because the Alfvén
speed is faster at the right than at the left of $x_0$, the wave fronts get tilted ($t_1$ in panel b), relative to the phase propagating with speed $V_{A0}$. At later time ($t_2$) the wave fronts get tilted even further and approach each other closely at the position $x_0$. This leads to small scale formation and hence to heating.

Consider now the situation in which the field disturbance is a wave with polar-
ization along the $y$ direction, $\delta B = B_y$, perpendicular to the field lines and the
direction of inhomogeneity (phase mixing, panel c in fig. 2.1). Since the local
Alfvén speed of adjacent field lines (at $x_1$ and $x_2$) is different, after propagating
some distance $\Delta z$ the fields $B_y(x_1)$ and $B_y(x_2)$ will be very different, leading to
a current sheet and strong dissipation.

The efficiency of resonant absorption has been studied mainly for close struc-
tures, such as coronal loops, in which waves are trapped. From the one hand
the energy of the fluctuation is hence localized in the structure and can be dis-
sipated easier, on the other hand, the dimension of the loop imposes a limit on
the maximum wavelength of the resonant mode, reducing the efficiency of res-
onant absorption (Malara & Velli 1994). Phase mixing have been extensively
studied in homogenous and inhomogeneous background equilibrium fields by
means of normal mode analysis and numerical simulations, in the incompress-
able limit, (Califano et al. 1990; Malara et al. 1992; Einaudi et al. 1993). The
resulting dissipation rate encouraged extensions to more realistic cases, includ-
ing compressibility (Califano et al. 1992; Ofman et al. 1994; Malara et al. 1996).

The coupling with compressible modes is efficient for oblique propagation and
results in an enhanced dissipation, even for small driving wave amplitudes,
caused by the formation of shock of the excited slow modes. These ones are a
by-product of the phase mixing, which does not directly imply the formation
of small scales, wave steepening being more efficient, and the energy dissipated
amounts to 15%-20% of that of the driving Alfvén wave. Since the simulation
had a low Reynolds number the applicability to the coronal heating problem
is limited, though the authors argued that efficient dissipation through mode
coupling should work at high Reynolds number as well, the steepening time
being independent from it.

Recently Nakariakov et al. (2000) studied the efficiency of phase mixing in
coronal holes (see also Ofman & Davila 1997). In particular they considered a
circularly polarized Alfvén wave propagating upward in a spherically diverg-
ing flux tube, allowing for both stratification and transverse inhomogeneity.
Such waves interact with the nonlinearly self-generated fast magneto-acustic
mode and produce a significant dissipation below say $10 R_\odot$. The dissipation
rate was found to be almost independent of the resistive coefficient adopted
while depends on the amplitude and period of the Alfvén wave: longer periods
or higher amplitudes increase the dissipation rate, decreasing the distance at
which the wave is efficiently dissipated (even below 3 $R_\odot$). However, the de-
pendence on the viscosity was investigated only in a small interval (one order
of magnitude) and the above conclusion is questionable. Moreover, the result
was derived under the assumption of small wavelengths $\lambda < H$, where $H$ is the
density scale height, limiting the applicability to wave periods of one minute or shorter. In contrast, Zaqarashvili & Belvedere (2005) show that low frequency Alfvén waves may be generated by fast magneto-acoustic waves through the swing-absorption, which works in the opposite direction of the above mechanism: energy is transferred from a compressible fast mode propagating across the mean magnetic field to the Alfvén waves propagating along the magnetic field line, which is parametrically amplified. The frequency of the amplified Alfvén waves is about the half of that one of driving fast mode, assuming that the solar oscillations in the radial direction are the driving source for the latter, a period of several hours is expected for the amplified Alfvén waves below the convection zone.

2.2.1 Solar Wind Models with Specific Heating Mechanisms

Fluid models with a prescribed ad-hoc form of the heating rate were generally successful in reproducing solar wind properties from the inner corona to the outer heliosphere, and definitively improved the understanding of the heating and acceleration of the solar wind from the energetic point of view. The major advances can be summarized in two main aspects which have been emphasized by Hansteen et al. (1999). First, the inclusion of the chromosphere in solar wind model and the build up of the transition region are of fundamental importance in treating the heating problem and acceleration problem on the same footing. It is hence necessary to include radiative losses, heat conduction and a heating function in the energy equation in order to achieve a fast solar wind. Second, the description must consider the multi-species characteristic of the solar wind, since electron, protons and ions all have a different role with respect to the different terms entering the energy equation.

However, such models tell us nothing about the specific mechanism which actually heats the solar wind and cannot be used to accept or refute the assumption that a given mechanism is the primary one in giving the right amount of heating and acceleration. For this purpose, solar wind models incorporating dissipation mechanisms must be constructed. This task is very challenging since one needs to couple large scale fluid quantities and small scale microphysics which describes the dissipation process. Hence, several simplifying assumptions are taken depending on the authors and on the process studied. In the following we will describe ion-cyclotron resonance, which is one of the most plausible mechanism emerged recently, together with some models which in different manner incorporate it in the fluid description.

Ion-Cyclotron Resonance

If one admits that the reservoir of energy are fluctuations at large scales, whatever mechanism is invoked to transfer the energy to small scales, one has to recognize that the usual description of dissipation in term of a viscous co-
2. The Fast Solar Wind, Heating and Acceleration Mechanisms

efficient is not adequate in treating plasma processes and true wave-particle interactions must be considered in order to model dissipation. UVCS results on anisotropic ions temperature and the preferential heating of heavy ions has focused the attention of the community to the ion-cyclotron resonance, even though direct evidence of its occurrence in the low corona has not been found yet.

To describe the ion-cyclotron resonance and the impact on solar wind models, we will follow the recent review on the subject by Hollweg & Isenberg (2002). Consider a transverse wave propagating along the mean magnetic field and a charged particle rotating around the field line at the gyro-frequency \( \Omega_i = q_i B / m_i c \), \( q_i \) and \( m_i \) being the charge and mass of the particle. When the electric field vector of the wave, in the reference frame of the particle, matches the cyclotron gyration of the particle, both in frequency and in the direction of rotation, the resonance occurs and the wave and the particle are able to exchanged energy efficiently. The resonance condition can be written as:

\[
\omega(k_{||}) - k_{||} v_{||} = \pm \Omega_i, \tag{2.6}
\]

where \( \omega \), \( k_{||} \) are the frequency and wavenumber of the wave parallel to the magnetic field and \( v_{||} \) is the parallel speed of the particle (the LHS is the doppler shifted frequency of the wave in reference frame of the particle). The \( \pm \) sign accounts for the sense of rotation of the wave electric field given by the wave polarization. For an Alfvén/ion-cyclotron wave the field rotates in the same direction of the ion (the sign is +) and resonance can be attained for slowly moving ions. For fast/whistler waves, the sense of rotation is the opposite and ion must have a speed larger than the wave phase speed to match the resonance condition (anomalous doppler shift). The dispersion relation \( \omega(k_{||}) \) of the wave considered determines whether or not a resonance may occur for a given species and for a given background magnetic field. To see what happens when resonance conditions is matched, consider a transverse circularly polarized wave propagating along \( B_0 \) taken to be in the \( z \) direction, its wavenumber and frequency being \( k_z \) and \( \omega \). An ion in resonance sees an electric field \( \delta E_\perp \) and gains energy at a rate \( q_0 \delta E_\perp v_{\perp,0} \cos \phi \) where \( v_{\perp,0} \) is the speed of the particle’s gyration transverse to \( B_0 \) and \( \phi \) the phase of gyration relative to the wave. Depending on the value of the phase, a particle can gain or lose energy. In the frame of reference of the particle the electric field is rotating at the particle’s gyrofrequency \( \Omega \) and the magnetic field can be written as \( \delta B_{\perp} = c k_z \delta E_\perp / \Omega \).

Finally the transverse energy of the particle increases according to

\[
v_{\perp,0} \delta v_{\perp} = v_{\perp,0} \frac{\Omega^2 t \delta B_{\perp}}{k_z B_0} \cos \phi \tag{2.7}
\]

and the Lorentz force along \( B_0 \) induces a parallel velocity gain

\[
\delta v_{\perp} = v_{\perp,0} \frac{\Omega t \delta b_{\perp}}{B_0} \cos \phi. \tag{2.8}
\]
In this derivation it was implicitly assumed that the wave is coherent, when instead resonance with an incoherent wave is considered, the above expressions depend on the power spectrum of the wave \( P_B \), defined by the relation

\[
\left\langle \delta B_\perp^2 \right\rangle = \int_0^\infty P_B(k_\perp) \, dk_\perp.
\]

Assuming a power spectrum for the fluctuation \( P_B \propto k^{-\gamma} \), with other few assumptions on the distribution function of the particles, one obtains a simplified form for the transverse heating rate, \( Q_\perp \), and the resonant acceleration, \( a_{\text{res}} \) (Isenberg & Hollweg 1983):

\[
Q_{\perp,\text{res}} \propto \left( \frac{m}{q} \right)^{\gamma-2} |V|^{\gamma+1}, \quad a_{\text{res}} \propto \left( \frac{m}{q} \right)^{\gamma-2} |V|^\gamma
\]

where \( V \) is the (negative) drift velocity of cold particles.

More than mass proportional heating requires a steep power spectrum \( \gamma > 2 \) if all the particles have the same \( |V| \). However, heavy ions resonate with faster-moving waves than do the protons, so generally they will have larger values of \( |V| \) than protons and one can have more than mass proportional heating of ions even with \( \gamma < 2 \) (Hollweg 1999a,b). For the same reason, i.e. resonance with faster moving waves, ions will be accelerated preferentially also for \( \gamma < 2 \). Hence these expressions assure a more than mass proportional heating and acceleration together with temperature anisotropies, as observed in the low corona (acceleration region). These “good properties” constitute the main reason for the enormous interest which has grown on the ion-cyclotron resonance mechanism.

**Solar Wind Model with Ion-Cyclotron Resonance**

The inclusion of ion-cyclotron resonance in solar wind, multi-species, fluid models is realized through the inclusion of a self-consistent description of Alfvén wave evolution. Isenberg & Hollweg (1982) and Isenberg (1984) were the first to give such a description derived using energy and momentum conservation, under the assumption of a steady wind, parallel magnetic field and bulk speed of the ions and small wavelength compared to the density scale height (WKB limit). In most of the models it was assumed that a turbulent cascade develops, energy being transferred to smallest scales where resonant processes irreversibly heat and accelerate the plasma. The resonance determines how energy is divided between protons and heavy ions, and between direct heating and work done by the resonant acceleration. The energy dissipation rate is determined by conditions at the large nonresonant spatial scale, which contains most of the energy. In the first models (Hollweg 1986; Hollweg & Johnson 1988; Isenberg 1990) a Kolmogorov or Kraichan dissipation rate derived from dimensional analysis was assumed and also a form of the power spectrum at the resonant range was prescribed (usually \( P_B \propto k_\parallel^{-\gamma} \) with \( \gamma = 5/3 \)). These two
fluid models where further refined, focusing on different aspect of the problem. Li et al. (1999a) included proton thermal anisotropy, Li et al. (1999b) incorporated the effect of dispersion in the resonant interaction which was found to enhance parallel proton cooling. Neglecting the temperature anisotropy, Hu et al. (1999) extended the concept of turbulent cascade including an equation which describes the evolution of a turbulent spectrum, as postulated by Tu et al. (1984) and Tu (1987, 1988). Hu & Habbal (1999) were the first to construct a three fluid model, including He$^{++}$, and treating in detail the effect of helium in the dispersion relation. They calculated the evolution of the turbulent cascade following Tu et al. (1984), but the specification of the spectral index in the dissipation range was again necessary (anisotropy was not considered). Hu et al. (2000) extended this model for the inclusion of a forth ion, the O$^{+5}$, which was well observed by UVCS, finding good agreement in both temperature and outflow velocity.

Although very few doubts remain on the fundamental role that ion-cyclotron resonance plays in the preferential heating of heavy ions and in the anisotropies of temperature, its importance with regard to the acceleration of the solar wind is uncertain, for different reasons. Recently, Isenberg (2004) have proved that taking into account properly the dispersion relation in a situation in which ion-cyclotron heating is maximized, solar wind properties, such as the outflow speed and temperature are not reproduced, the acceleration exerted on protons is too low and resonant forces act in the direction of perpendicular cooling, rather than heating. Oblique propagation, however, can not be modeled in the adopted formalism and could produce interesting results.

The other remark, which applies to the solar wind model mentioned above, concerns the development of the turbulent cascade, which has been the primary mechanism invoked to drive the energy to the small resonant scales. The wave equations in multi-species fluid has in fact been derived under the assumption of small wavelength, which contradicts the fact that turbulence develops, nonlinear interactions occurring only among counter-propagating waves\(^1\). This difficulty was circumvented prescribing a constant (or an ad hoc varying) relative population of inward and outward propagating waves which may be a good approximation in the outer corona but is questionable for regions close to the Sun, where the acceleration of the solar wind takes place (see Axford & McKenzie 1992; Sturrock 1999 for a different point of view in which energy is injected directly at high frequencies at which the ion-cyclotron resonance operates, an hence a cascade is not needed). Even in the non-WKB limit, the smallness of the wave amplitudes poses a serious doubt in the actual development of a turbulent cascade, which needs detailed investigation (a topic which is partially addressed in this thesis). Moreover, as will be discussed in the next section, MHD turbulence is efficient in transferring energy in the perpendicular

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\(^1\)This will be clear in the next session
length scales, but the cascade in the parallel wavenumber (or in frequency) is suppressed because of the strong parallel magnetic field.

### 2.3 Turbulence in the Fast Solar Wind

As the solar wind expands in the heliosphere, nonlinearities develop a strong turbulent state which, in its final evolution, resembles the hydrodynamic turbulence described by Kolmogorov (1941), despite its state is that of a plasma threatened by a strong magnetic field. The description of the large scale properties of the solar wind, and the turbulence which develops, is usually made in the contest of a magnetohydrodynamic description because of the low frequency of the fluctuations, even though, as discussed before, kinetic physics must be considered when small scales are involved. The large range of scales, that one has to consider in studies of turbulence, is usually described by means of fluid description and the microphysics which determines the dissipative process is neglected in favour of viscous and resistive coefficients. Turbulence in the heliosphere influences the plasma behavior in many aspects beyond the solar wind acceleration problem, such as the high energy particle acceleration and the scattering of cosmic rays. A detailed description of turbulence requires in-situ observations, which in the 70s and 80s were limited to the ecliptic plane. With the launch of Ulysses the investigation has been extended to high latitude regions of the heliosphere, allowing to study turbulence in the polar solar wind. Helios and Ulysses measurements cover a wide range of heliocentric distances, from as close as 0.29 AU (Helios) to the distances of 9 AU (Ulysses). Unfortunately closest approach to the Sun are not in future plans and will not help in revealing the early stage of the solar wind turbulence, i.e. its spectrum close to the Sun, which would help in solving also the acceleration problem. A spacecraft, however, covers a small area compared to the scale characteristic of turbulent phenomena in the solar wind, only one point-measurements have been possible, in contrast to two-point correlation measurements which best describe turbulent phenomena, both observationally and theoretically. Concerning this issue, the STEREO mission will offer the opportunity of such two-point measurements during its first part of the orbit, while the four CLUSTER spacecrafts, around the Earth, are providing us a three dimensional description of the turbulence (although limited in its spatial extent) which develops in the encounter of the solar wind with the Earth magnetosphere.

#### 2.3.1 Turbulence Phenomenology

The basic properties of turbulence can be derived from the Navier-Stokes equation (in the fluid case) or the MHD equations (in the magnetized case) or from phenomenological considerations. The “simplest” situation, in both cases, is obtained assuming incompressibility, which yields the following equations for
the velocity \( u \) and magnetic \( B \) field:

\[
\frac{\partial u}{\partial t} + u \cdot \nabla u = -\nabla p + \nu \nabla^2 u + B \cdot \nabla B + f \tag{2.11}
\]

\[
\frac{\partial B}{\partial t} + u \cdot \nabla B = B \cdot \nabla u + \eta \nabla^2 B \tag{2.12}
\]

where \( p \) is the pressure, \( \nu \) and \( \eta \) are the kinematic viscosity and magnetic diffusivity respectively, and \( f \) is the force which models the large scale energy input. The specific energy injection mechanism may vary widely in different astrophysical context, but it is assumed that the scale at which it operates, say \( L \), is large comparable with the size of the system. While the large scale dynamics depends on the injection mechanism, at scales much smaller then \( L \) the nonlinear dynamics is universal. This assumption of universality is fundamental in all turbulence theories.

The dimensional derivation of the turbulence phenomenology is outlined in the following. Consider eq. 2.11 without the magnetic field and let \( \delta u_L \) be the typical fluctuating velocity difference across the scale \( L \). The energy associated to this fluctuation and the characteristic timescale for this energy to cascade to smaller scales are given by \( \delta u_L^2 \) and \( L/\delta u_L \). The energy flux going into the turbulent cascade is given by \( \epsilon = \langle u \cdot f \rangle \approx \delta u_L^3 L/\nu \), where angle brackets denote a spatial (or an ensemble, or a time) averaging procedure. In a statistically stationary situation all the power must be dissipated so that \( \epsilon = \nu \langle |\nabla u|^2 \rangle \). A characterization of turbulent motions is that in the limit of vanishing viscosity the dissipation attains a constant, finite value. In other words, for small \( \nu \) this implies that the velocity must develop very small scales and \( \nu \langle |\nabla u|^2 \rangle \) has a finite limit for \( \nu \to 0 \). The only quantity with dimension of a length that can be constructed with \( \epsilon \) and \( \nu \) is \( l_\nu \approx (\nu^3/\epsilon)^{1/4} \approx \text{Re}^{-3/4} L \), which gives the expression for the above small scales. Here \( \text{Re} = \delta u_L L/\nu \) is the Reynolds number. In astrophysical situations \( \text{Re} \) is very high and viscous dissipation occurs at scales \( l_\nu << L \). The energy injected at large scale \( L \) must be transferred to small scale \( l_\nu \) across a range of scales, the inertial range. The universality assumption implies that the physics in this range is independent of both the injection and the dissipation mechanisms. Further assumptions must be taken which describe the dynamics in the inertial range, namely homogeneity, scale invariance\(^2\), isotropy and locality of interactions. The latter means that nonlinear interactions are dominated by interactions between comparable scales. At each scale \( l \) in the inertial range \( L >> l >> l_\nu \) the energy arrives from large scales and is transferred to smaller scales at the same rate

\[
\epsilon \approx \delta u_l^2/\tau_l, \tag{2.13}
\]

where \( \tau_l \) is the cascade time and \( \delta u_l \) the fluctuating velocity difference at the scale \( l \), which under the assumption of locality, can only be used to construct

\(^2\)a property of the Navier Stockes equation in which viscosity is neglected
the cascade time: \( \tau_l \approx l/\delta u_l \). Substituting in eq. 2.13 we get the Kolmogorov scaling, \( \delta u_l \approx (\epsilon l)^{1/3} \). Finally one obtains the energy spectrum \( E(k) \):

\[
\delta u_l^2 \approx \int_{k=1/l}^\infty dk' E(k') \approx e^{2/3}k^{-2/3} \quad \Rightarrow \quad E(k) \approx e^{2/3}k^{-5/3}, \tag{2.14}
\]

which gives the power low scaling for Kolmogorov turbulence.

**Alfvénic Turbulence** Consider now a plasma threatened by a straight uniform background field \( B_0 \), in the incompressible case (\( \rho \) is uniform). Let’s restrict the description to weak forcing, so that the turbulent excitations may be considered of small amplitudes and isolate the fluctuating part \( \delta B \) of the magnetic field as follow, \( B = B_0 + \delta B \). In term of the Elsässer variables \( Z^\pm = u \mp \text{sign}(B_0)\delta B/\sqrt{4\pi \rho} \), eqs. 2.11-2.12 may be rewritten as:

\[
\frac{\partial Z^\pm}{\partial t} \pm V_a \cdot \nabla Z^\pm = -\frac{\nabla p_{\text{tot}}}{\rho} - Z^\mp \cdot Z^\pm + \frac{\nu + \eta}{2}\nabla^2 Z^\pm + \frac{\nu - \eta}{2}\nabla^2 Z^\mp + f \tag{2.15}
\]

with \( V_a = B_0/\sqrt{4\pi \rho} \) (the Alfvén speed). \( Z^+ \) and \( Z^- \) are two exact solutions of the equations above and their polarization is either that of slow waves or that of Alfvén waves propagating in opposite direction (the latters are exact solution relaxing the assumption of small amplitudes as well; in the following, consider only Alfvén waves, for simplicity). Note that nonlinear interactions occur only among counter-propagating fluctuations.

The characteristic time entering eqs. 2.15 are the Alfvén time \( \tau_a \approx l/B_0 \) and the eddy turnover time, the lifetime of a vortex, \( \tau_{nl}^+ \approx l/z^+ \). In an encounter of two vortices, the variation in the amplitudes \( dz^\pm \) during the interaction time \( \tau_i \) is given by \( dz^\pm \approx z^\pm z^\mp \tau_i/l \), where \( z^\pm \) is the fluctuation amplitude at scale \( l \). The interaction time is the shorter between \( \tau_a \) and \( \tau_{nl}^+ \). For small amplitude fluctuations, \( dz^\pm \ll z^\pm \), the nonlinear interaction among two modes is reduced by the decorrelation effect (an exponential modulation which averages to zero for increasing \( V_a \)). Assuming the process stochastic, in \( N \) interactions the total amplitude variation will be \( \Delta z^\pm \approx \sqrt{N}dz^\pm \). The eddy turnover time, or cascade time is

\[
T^\pm \approx \frac{(\tau_{nl}^+)^2}{\tau_i}, \tag{2.16}
\]

and the corresponding number of interaction is \( N^\pm \approx (\tau_i^{-1}l/z^\mp)^2 \). If \( \tau_a \ll \tau_{nl}^+ \) (strong magnetic field) the energy flux is given by

\[
\epsilon \approx \frac{(z^\pm)^2}{T^\pm} = \frac{(z^\pm)^2 z^\mp \tau_a}{l \tau_{nl}^+} \tag{2.17}
\]

If the counter-propagating waves have comparable amplitudes \( \delta z \approx z^+ \approx z^- \) one gets:

\[
\delta z \approx (eIV_a)^{1/4} \quad \Rightarrow \quad E(k) \approx (eIV_a)^{1/2}k^{-3/2} \tag{2.18}
\]

the Iroshnikov-Kraichnan (IK) spectrum (Kraichnan 1964; Iroshnikov 1964).
Anisotropic Turbulence  Simulations and observations in the solar wind have shown that the turbulence is strongly anisotropic, with $I_\perp \ll I_\parallel$ and that nonlinear interactions are mediated by waves with low parallel wavenumber (resonant interactions). For a three wave interaction the wavenumber of the other two modes does not change, the cascade proceeding mainly in the directions perpendicular to the magnetic field and one can assume $I_\parallel \approx 1/k_{||0} = \text{const}$ (the wavenumber at which the waves are launched). Since now $I = I_\perp$ and $\tau_a = I_\parallel/V_a$, eq. 2.17, for fluctuations of comparable amplitudes, becomes:

$$\epsilon \approx \frac{\delta z^3}{I_\perp V_a I_\parallel},$$

and finally the energy spectrum follows a different power-law scaling:

$$\delta z \approx (\epsilon k_{||0} V_a)^{1/4} k_\perp^{1/2} \Rightarrow E(k) \approx (\epsilon V_a k_0)^{1/2} k_\perp^{-2}. \quad (2.20)$$

The assumption of weak interactions $\tau_a \ll \tau_{nl}$ does not hold at all scales, indeed from the scaling law of the amplitude fluctuation eq. 2.20 it follows that

$$\frac{\tau_a}{\tau_{nl}} \ll 1 \iff I_\perp >> l_* = \frac{\epsilon^{1/2}}{(k_{||0} V_a)^{3/2}} \approx \frac{\delta z_{L}^2}{V_a^2 k_{||0}^2 L}, \quad (2.21)$$

where $\delta z_{L}$ is the rms amplitude at the outer scale. In the solar wind (and many other astrophysical situations) the kinetic and magnetic Reynolds numbers, $Re$ and $Rm = \delta BL/\eta$, are very large and the inertial range will contain a scale $l_*$ below which the interactions are no longer weak. Below this limit Goldreich & Sridhar (1995) developed a theory (strong turbulence) in which $\tau_a \approx \tau_{nl}$ and $\delta z/V_a I_||/I_\perp \approx 1$ (critical balance). According to these relations the spectrum in the perpendicular wavenumber follows a Kolmogorov power low scaling. At this scales also a parallel cascade develops with $l_\parallel \approx V_a e^{-1/3} l_\perp^{2/3} \approx k_{||0}^{-1} (l_\perp/l_*)^{2/3}$. We will not discuss this interesting regime since it involves scales that will not be resolved in our simulations. However, the recover of a parallel cascade opens an entire field of research for the verification of the assumption, commonly adopted in solar wind model, that turbulence is able to transfer energy to small temporal scales at which kinetic (dissipative) processes are at work.

2.3.2 Properties of the Solar Wind Turbulence

The first evidence of the turbulent character of the fluctuations in the solar wind was given by Coleman (1968). He found that the magnetic fluctuations extended over a wide range of frequencies, from period corresponding to one cycle rotation to a few seconds. The energy spectrum follows a power-law scaling with slope approximately $f^{-1.2}$, being neither of Kolmogorv nor Kraichnan type. Using data from Mariner 2, Mariner 4 and OGO 5, Russell (1972) gave a composite picture of the radial component of the magnetic fluctuations identifying three ranges with a different power-law behavior: at low frequencies (up
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2.3. Turbulence in the Fast Solar Wind

Figure 2.2: Power density spectra of magnetic field fluctuations observed by Helios 2 in the range between 0.3 AU and 1 AU. The spectral break (blue dot) moves to lower frequencies with increasing heliocentric distance (after Bruno & Carbone 2005).

Helios allowed to follow the radial evolution of the magnetic spectra, between 0.29 AU and 1 AU, measuring the properties of the plasma which belongs to the same corotating stream. As shown in fig. 2.2, all the spectra are characterized by two main spectral components: low frequencies have a $f^{-1}$ behavior, then a break occurs and the slope becomes Kolmogorov-like at high frequencies, $f^{-5/3}$.

The break moves to lower frequencies as the heliocentric distance increases: as the wind expands large scales enter the Kolmogorov-like turbulent spectrum that is usually identified with the inertial range. Hence the power spectrum of the fluctuation depends on the frequency and on the distance, $P = P(f, r)$.

Belcher & Davis Jr (1971) showed that at 1 AU, in the intermediate frequency range $10^{-4} \text{ Hz} < f < 10^{-2} \text{ Hz}$, the magnetic and velocity fluctuations present a strong correlation, $\delta v \approx \pm \delta B / \sqrt{4 \pi \rho}$, with the sign given by the direction of the magnetic field (+ for $B_0$ pointing toward the Sun), the magnetic field magnitude is almost constant and density fluctuations are very small, suggesting incompressibility. Such fluctuations resemble Alfvén waves propagating outward from the Sun and are found to be ubiquitous in the solar wind. The early measurements of the anisotropies of the magnetic and velocity fluctuations...
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(1971) were successively refined adopting the technique of the minimum variance analysis (1967). This method briefly consists in studying the properties of the fluctuations in the minimum variance reference frame which is determined by the eigenvectors of the matrix

\[ S_{ij} = \langle B_i B_j \rangle - \langle B_i \rangle \langle B_j \rangle, \]

where \( i \) and \( j \) denote the components of the magnetic field in a given reference frame. For Alfvénic turbulence, one of the eigenvalues of \( S_{ij} \) is always much smaller than the others and the corresponding eigenvector identifies the minimum variance direction, which is on average parallel to the mean magnetic field \( B_0 \) with a spread of \( 10^\circ \). The power of the fluctuations in this direction is much smaller than the power in the other direction, showing up the anisotropies of the fluctuations (mostly found in the high velocity streams, 1990). In fast streams the direction of minimum variance for the velocity and the magnetic fluctuations are close to each other and both localized around the direction of the mean magnetic field (1991, 1993). Not only the fluctuations are mainly perpendicular to the magnetic field but also the correlation length is anisotropic, being larger in the direction perpendicular to the mean magnetic field. Measurements in the ecliptic, mainly for the slow solar wind, suggested a 2D turbulence (1990), while polar measurements (2003) suggested a strong presence of slab turbulence in the fast streams.

A measure of the Alfvénicity of the fluctuations is given by the normalized cross helicity

\[ \sigma_c = \frac{2 \Re \{ u \cdot b \}}{(u^2 + b^2)}, \]

and the Alfvén ratio

\[ r_A = \frac{e_u}{e_b}, \]

where \( b \) is expressed in unit of velocity and \( e_u = 1/2 |u|^2 \), \( e_b = 1/2 |b|^2 \) are the energies per unit mass in the velocity and magnetic fluctuations. The former can be also interpreted as the relative amount of outward and inward propagating fluctuations, for strong correlation (Alfvénicity) \( \sigma_c = 1 \) (only outward fluctuations) and \( r_A = 1 \), which means equipartition. While phenomenology of MHD turbulence, as outlined in sec. 2.3.1, usually assumes \( \sigma_c = 0 \) (equal amount of \( \sigma^+ \) and \( \sigma^- \)) as a condition for a balanced energy transfer in the two species, in the solar wind the situation is more complex. Close to the Sun, fluctuations are more Alfvénic at low frequencies (\( \sigma_c \approx 1 \)) while the degree of correlation decreases with increasing frequency (approaching \( \sigma_c \approx 0.5 \) for \( T \approx 3 \) h). The radial evolution shows that the general trend is that of a decreasing correlation, both low and high frequencies attain a value of cross helicity close to zero at about 20 AU (1987).

The Alfvénic character of the fluctuations stimulated theoretical and observational investigations based on the separation among the outward and inward components of the Alfvénic turbulence, i.e. an analysis relaying on the Elsässer variables (see sec. 2.3.1). The power densities of the outward and inward fluctuations \( (e^\pm) \) show variations related to the wind speed and distance. While in the slow wind the power densities have almost the same level, in the fast wind there is a dominance of the outward component. In the fast streams, the inward power spectrum maintains the same shape, with a slope of about \(-1.64 \) at different distances (1990), while the outward spectrum...
2.3. Turbulence in the Fast Solar Wind

Figure 2.3: Trace of $e^+$ (solid line) and $e^-$ (dash-dotted line) power spectra. The central and right panels refer to Ulysses measurements (in the fast south polar wind) and the left panel refers to Helios measurements (crossing slow and fast streams) (after Goldstein et al. 1995).

is somewhat flatter at low frequencies and steeper at high frequencies close to the Sun. Its overall evolution with increasing heliocentric distance is toward the same slope of the inward spectrum (maintaining its higher level). The evolution with distance of the cross helicity can be imputed to the variation of the energy residing in the outgoing mode rather than to the variation of the energy in both modes. Indeed, measurements between 1 AU and 5 AU shows that the ratio $r_E = e^-/e^+$ (Elsässer ratio) approaches a constant value of about 0.3 in contrast with theoretical studies which indicate that if an imbalance exists among the two modes, the evolution of turbulence is toward a state in which only the dominant mode is present (dynamic alignment, i.e. maximum correlation). In the distant solar wind the inward component must be locally produced to maintain such a saturated level (by shear interaction among fast and slow stream probably).

With the polar transit of Ulysses, the properties of high latitude solar wind became accessible to observations. The polar Alfvénic turbulence is found to evolve in the same way as in the ecliptic plane, but more slowly. The absence of strong velocity shear (at solar minimum) favors the persistence of the Alfvén correlation at low frequencies, the turbulence is less “aged”, but still active, so that some other mechanisms should be invoked (such as parametric instability) at large heliocentric distance, in order to sustain turbulence through the local production of the inward component, necessary for nonlinear interactions to occur. The spectra of magnetic fluctuations resemble the ecliptic counterpart, being flat at low frequencies and Kolmogorov-type at high frequencies (Smith et al. 1995). The Elsässer spectra taken at 2 AU and 4 AU (shown in fig. 2.3) suggest the above mentioned slower evolution compared to Helios results. Ulysses observations at 2 AU resemble more the turbulence conditions...
observed by Helios at 0.9 AU rather than at 0.3 AU also for what concerns the level of cross helicity: $\sigma_c \approx 1$ at high frequencies ($\gtrsim 10^{-5}$ Hz) while it decreases toward zero at low frequencies.

The radial evolution of the Elsässer energies at high frequencies is commonly imputed to the nonlinear interactions (turbulence) among Alfvénic fluctuations which are locally generated in the solar wind (by parametric instability, shear interaction, pick-up ions), since a strong correlation among the energy levels of $\delta z^+$ and $\delta z^-$ persists and the Elsässer ratio is about 0.2. On the contrary, at low frequencies the fluctuations of $z^+$ and $z^-$ appear to be uncorrelated. The strong Alfvénicity of these fluctuations ($r_E \approx 10^{-2}$, Marsch & Tu 1990) is in contrast to the manifested nonlinear behavior (evolving spectra), while the outward component is of solar origin, the nature of the inward component is still unclear. The $\delta z^-$ could be imputed to either structures advected by the wind expansion (Pressure Balanced Structures or Tangential Discontinuity) or low frequency waves, hence of Alfvénic nature. These are reflected by the inhomogeneity of the background wind in the inner heliosphere and advected by the wind expansion which freezes their nonlinear evolution (usually referred as 2D fluctuations).

The radial dependence of the hourly averaged Elsässer energies (the Alfvénic spectrum) in the distant solar wind (from 1.4 AU to 4.3 AU) is shown in fig. 2.4 (Bavassano et al. 2000b,a). Inside 2.5 AU the energy in the outward mode decreases faster than that of the inward mode, with a slope consistent with Helios measurements. Beyond this distance, the slope of the $e^-$ energy becomes steeper while that one of $e^+$ is almost unchanged. Note that the decrease for $e^+$
is faster than what is predicted by the WKB approximation \((\propto r^{-1})\). The net change at 2.5 AU is clearly visible in the variation of the Elsässer and Alfvén ratio. The latter settles around a value of 0.25 and oscillates slightly, indicating that the magnetic energy largely dominates at great distances. From the values of the cross helicity and residual energy, \(\sigma_d = (e_u - e_b)/(e_u + e_b)\), at different heliocentric latitudes, it results, as a general feature, a predominance of outward modes and high magnetic field \((\sigma_c > 0 \text{ and } \sigma_d < 0)\) and that the Alfvénic spectrum is localized at high latitude and short heliocentric distance. The occurrence of a large number of event presenting an equal amount of outward and inward modes and a strong magnetic dominance \((\sigma_c \approx 0 \text{ and } \sigma_d \approx -1)\) was interpreted Bavassano et al. (1998) as a possible signature for the presence of quasi 2D fluctuations.

### 2.3.3 Alfvén Waves and Turbulence in the Inhomogeneous Solar Wind

We have seen that in the distant solar wind (beyond say 1 AU) many processes are involved in the sustainment of the turbulent dynamics: shear interaction in the ecliptic, parametric instability in both the polar and ecliptic region and pick-up ions in the outer heliosphere. In that regions the solar wind has already attained its cruise speed and can be regarded as a uniform, radial flow. The magnetic field, in the configuration of the Parker spiral, deviates from the radial direction and the angle formed with the radial increases with heliocentric distance (it is about 45° at 1 AU).

In the corona the wind experiences a strong acceleration and the rapid decrease of density and magnetic field produce a strong variation of the Alfvén speed. A wave propagating in the solar wind sees a background equilibrium which is hence more inhomogeneous and, more dramatically in the inner layers of the solar atmosphere. Assumed that the outward propagating fluctuations observed in the fast streams are of solar origin, it is worthwhile to study their propagation properties in the inner atmosphere. This waves can be regarded as small amplitude perturbations below a certain distance (a few \(R_S\) from the solar surface) but in the corona their amplitude grows due to the variation of their group velocity \((V_g = U + V_a)\) and nonlinear interaction, if any, cannot be neglected. The inhomogeneity has also the effect of reflecting the freely propagating wave: as a flux of outward waves is launched from the photosphere an inward flux is created by this mechanism, the stronger the variation of \(V_g\) the highest the coupling of the two modes. This mechanism of generation of counter-propagating waves is necessary for the triggering of on nonlinear interactions (a schematic representation is given in fig. 2.5). Since the Reynolds number is very high in the solar atmosphere, such nonlinear interactions can develop a turbulent state in which the energy cascade to small scales, finally heating the plasma. This mechanism could in principle account for the generation of the turbulent cascade invoked in many wind models that include the
2. The Fast Solar Wind, Heating and Acceleration Mechanisms

Figure 2.5: Schematic representation of the generation of turbulence through the reflection of outward propagating waves in a coronal hole. The waves are generated at the base of the photosphere by the shaking of the footpoint of the magnetic field line induced by the photospheric motions. As the waves propagate upward, variation in the propagation speed (induced mainly by density gradients) produces a flux of downward reflected waves that interacts nonlinearly with the waves escaping from below. Since the Reynolds number are very high, for large enough wave amplitudes, successive nonlinear interactions generate a turbulent cascade which may lead to an efficient dissipation.

Nonlinear Interactions

Reflection

\( \delta u \)

The equations describing the evolution of Alfvénic fluctuations in the solar wind can be derived from a timescale separation of the full MHD equations and with a few further assumptions (incompressible, transverse fluctuations) as an acceleration mechanism and could reproduce the Alfvénic turbulence observed at short heliocentric distances by Helios. In order to ascertain its relevance relative to the above issues the development of the (incompressible) turbulence must be tested in solar-like conditions, then the resulting heating rate can be compared to the requirements of solar wind models, and finally the evolution of an initial spectrum of fluctuations must be followed in the expanding wind. Linear and nonlinear effects on the evolution of the Alfvénic fluctuations must be considered in detail and possibly on the same footing, since the inhomogeneity and the wave amplitudes vary largely from the photosphere to the solar wind. In the following we briefly review the properties of Alfvén waves propagating in an inhomogeneous stationary moving medium in order to clarify the reflection mechanism and quantify the importance of linear effects on the evolution of the Alfvén spectrum.
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Assume a slow and fast timescale separation of the mean fields, $B_{\text{tot}} = B + b$, $U_{\text{tot}} = U + u$ while the density does not vary because of the incompressibility of the fluctuations, $\rho_{\text{tot}} = \rho$. The continuity, momentum and induction equations are:\footnote{The derivation of these equations is in the appendix A}

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho U) = 0, \tag{2.22}
\]

\[
\rho \left( \frac{\partial U}{\partial t} + U \cdot \nabla U \right) = -\frac{GM_\odot \rho}{r^2} \hat{r} - \nabla \left( p + \frac{(b^2)}{8\pi} \right) + \frac{1}{4\pi} B \cdot \nabla B + \left( \frac{1}{4\pi} b \cdot \nabla b - \rho u \cdot \nabla u \right). \tag{2.23}
\]

\[
\frac{\partial B}{\partial t} + U \cdot \nabla B = B \cdot \nabla U - (\nabla \cdot U) B + \langle b \cdot \nabla u - u \cdot \nabla b \rangle. \tag{2.24}
\]

For the energy equation, including only the turbulent dissipation $Q$, one has (heat conduction and radiative losses are neglected):

\[
\frac{\partial p}{\partial t} + U \cdot \nabla p = -\gamma (\nabla \cdot U) p + (\gamma - 1) Q. \tag{2.25}
\]

The velocity and magnetic field fluctuations are conveniently written in terms of the Elsässer variables $Z^\pm = u \mp \text{sign}(B) b / \sqrt{4\pi \rho}$, which represent Alfvén waves propagating outward (+) from the Sun and inward (−) for both the polarization of the mean magnetic field (pointing outward or inward). Assuming steadiness of the mean fields above and colinearity between the magnetic and gravitational fields, one obtains:

\[
\frac{\partial Z^\pm}{\partial t} + [(U \pm V_a) \cdot \nabla] Z^\pm + (Z^\pm \cdot \nabla) (U \mp V_a) + \frac{1}{2} (Z^+ - Z^-) \nabla \cdot \left( V_a \mp \frac{1}{2} U \right) = -\frac{1}{\rho} \nabla p_{\text{tot}} - [(Z^\pm \cdot \nabla) Z^\pm - \langle (Z^\pm \cdot \nabla) Z^\pm \rangle], \tag{2.26}
\]

where the average Alfvén speed is $V_a = B / \sqrt{4\pi \rho}$. On the right hand side we have grouped the nonlinear terms including total (magnetic plus gas) pressure, which may be written as the product between $Z^+$ and the gradients of $Z^-$ and viceversa. The nonlinear terms which don’t average to zero (in angle parenthesis) are to be considered part of the background medium equation, hence must be subtracted in the fluctuation equations. In the linear part of the equation we recognize a propagation term (II) and two terms accounting for reflection due to the variation of the properties of the medium, one isotropic (IV) while the other (III) involves variations along the fluctuations’ polarization.

\[\text{t} \text{:.} \text{u} \text{r} \text{b} \text{l} \text{u} \text{v} \text{e} \text{n} \text{t} \text{i} \text{u} \text{r} \text{a} \text{t} \text{e} \text{s} \text{n} \text{.} \text{A} \text{p} \text{p} \text{e} \text{n} \text{d} \text{i} \text{x} \text{i} \text{a} \text{p} \text{e} \text{n} \text{d} \text{i} \text{c} \text{t} \text{A}
\]
Linear Analysis and Applications

Since the ambient fields are assumed to be stationary the Elsässer fields can be Fourier decomposed in time, each component being labelled with its frequency $\omega (\mathbf{r}) \exp[-i\omega t]$. In a static atmosphere one finds the conservation of the wave energy flux,

$$\nabla \cdot \left[ \rho v_a \left( |z^+|^2 - |z^-|^2 \right) \right] = 0,$$

(2.27)

while when a non-uniform moving medium is considered the conservation applies to the total wave action density (wad). The wad is defined as the ratio of the wave energy density and its intrinsic frequency, i.e. the doppler shifted frequency in the reference frame comoving with the mean flow. Adding the fluxes associated to both waves at a given frequency one gets (Heinemann & Olbert 1980; Barkhudarov 1991; Velli et al. 1991):

$$\nabla \cdot \left\{ \frac{\rho}{v_a} \left[ (U + v_a)|z^+|^2 (U + v_a) - (U - v_a)|z^-|^2 (U - v_a) \right] \right\} = 0.$$

(2.28)

The two distinct fluxes, associated with downward ($S^- \propto U - v_a$) and upward ($S^+ \propto U + v_a$) propagation, in the case of radial propagation reduce to

$$S^\pm = F \frac{(U \pm v_a)^2}{UV_a} |z^\pm|^2,$$

(2.29)

where $F = \rho UR^2$ stands for the total mass flux, also conserved in virtue of the continuity equation; the conservation eq. (2.28) thus reduces to the statement $S^+ - S^- = S^* = \text{const.}$

At the Alfvénic critical point ($X_a$ where $U = v_a$), the inward flux of wave action density vanishes ($S^+_a \equiv S^*$) and one can impose the energies of both counter-propagating waves (see below). Given a layer of atmosphere with top boundary at $X_a$ it is possible to define a transmission ($T$) and a reflection ($R$) coefficient:

$$T = \frac{S^+_0}{S^+_0} \equiv \frac{S^+}{S^-} \quad \text{and} \quad R = 1 - T = \frac{S^-}{S^-},$$

(2.30)

where $S^+_0, S^-_0$ are the fluxes at the bottom boundary.

Linear Model Equations We furthermore reduce the complexity of the basic equations neglecting the dependence from the perpendicular coordinates (and hence the dynamics in the perpendicular plane). All the variables depend only on the scalar radial coordinate ($r$, spherical symmetry), however we allow the flux tube expansion to be nonradial, so that an area expansion

\footnote{With capital Z we refer to the frequency integrated Elsässer variables, for the fourier components we use $z$}
factor $A(r)$ can be prescribed (which in spherical symmetry is simply $A = r^2$). Expressing velocities in unit of $u^*$, lengths in unit of $R^*$ and frequencies in unit of $1/t^* = u^*/R^*$, the dimensionless form of eq. (2.26), linearized and reduced for the spherical symmetry case, finally becomes:  

$$\frac{dz^\pm}{dr} - i \frac{\omega}{U \pm V_a} z^\pm + \frac{U \mp V_a}{r(U \pm V_a)} z^\pm + \frac{1}{2} \frac{z^- - z^+}{U \pm V_a} \left( \frac{1}{A} \frac{dA}{dr} + \frac{d}{dr} \right) \left( V_a \mp \frac{1}{2} U \right) = 0.$$  

(2.31)

The Alfvén critical point represents a regular singularity for eq. (2.31). The regularity conditions for the solutions at $X_a$,

$$\Re (z^-_a) = \frac{\nu u}{\mu^2 + \omega^2} \Re (z^+_a) + \frac{\nu \omega}{\mu^2 + \omega^2} \Im (z^+_a),$$

(2.32)

$$\Im (z^-_a) = \frac{\nu \omega}{\mu^2 + \omega^2} \Re (z^+_a) + \frac{\nu u}{\mu^2 + \omega^2} \Im (z^+_a),$$

(2.33)

can be used to impose boundary conditions. The coefficients $\mu$ and $\nu$ are functions of the Alfvén and wind speed (and their derivatives) calculated at the Alfvénic critical point. If all the variables dependence is reduced to $r$ (instead of $r$), they can be written as:

$$\mu = \frac{1}{2} \left( \frac{dU}{dr} + \frac{1}{A} \frac{dA}{dr} U \right)_{r=X_a}, \quad \nu = \frac{1}{2} \left( \frac{dU}{dr} - \frac{1}{A} \frac{dA}{dr} U \right)_{r=X_a},$$

(2.34)

and their values are of the same order of magnitude if the Alfvénic critical point is far enough from the atmospheric base. Since eq. 2.28 is independent of the wave phase one can choose $|z_a| = \Re (z^+_a) = z_a^+$. It follows that imposing the same amplitude for all the frequencies results in an total energy density which is decreasing with frequency:

$$\epsilon_a = \frac{1}{4} \nu_2 \left( |z_a^1|^2 + |z_a^2|^2 \right) = \frac{1}{4} \nu_2 |z_a^1|^2 \left( 1 + \frac{\nu^2}{\mu^2 + \omega^2} \right).$$

(2.35)

Adopting a change of variable, eq. 2.31 can be furthermore simplified. Recalling that the Alfvénic Mach number $M_a = U/V_a \propto n^{-1/2}$, and that the wave equations in a static atmosphere are recovered in the limit of vanishing $M_a$, the following normalization ($O$, $N$ stand for old and new variables respectively)

$$z^\pm_N = \frac{M_a \pm 1}{\sqrt{M_a}} z^\pm_O \quad \left[ z^\pm_N = \pm r^{1/4} z^\pm_O \quad \text{for the static case} \right],$$

(2.36)

removes the systematic variation of the wave amplitude associated to its WKB behavior (the diagonal reflection terms$^6$, leading to the form:

$$\frac{dz^\pm_N}{dr} - i \frac{\omega}{U \pm V_a} z^\pm_N - \frac{1}{2} \frac{V_r}{V_a} z^\pm_N = 0,$$

(2.37)

for atmospheres with wind.

---

$^5$ $u^*$ and $R^*$ will be chosen depending on the characteristics large scale values of the problem, i.e. on the atmosphere

$^6$ Actually the change of variable is valid far from $X_a$, the transformation becoming singular at that point. When the critical point is included in the domain of interest eq. 2.31 is numerically integrated rather than its reduced form eq. 2.37.
The second and third coefficient in both the equations represent the propagation (P) and reflection (R) coefficients respectively (inverse of parallel wavelength, reflection scale height) and can be used to analyze the wave propagation properties.

**Results from a Linear Analysis** In the linear case, the atmosphere (with wind or not) behave as a frequency filter for a flux of outward propagating waves injected at the bottom of the domain. The characterization of the frequency filtering can be determined by means of the transmission coefficient, eq. 2.30, imposing the correct boundary conditions (vanishing inward flux at the border of the layer considered). The case of a spherically symmetric isothermal atmosphere (with wind or not) has been analyzed by Velli (1993) for low values of the plasma $\beta$ (as found in the solar corona). Generally, the local reflection rate is greater for low frequency waves and it decreases with frequency. The transmission coefficient is instead an “integrated” measure of the reflection in the whole layer and shows different properties. For temperatures of about $10^6$ K, high-frequency waves ($\omega \gtrsim 10^{-3}$ Hz) are completely transmitted. Decreasing the frequency, transmission decreases (i.e. reflection increases) and reaches a minimum at about $10^{-4}$ Hz. For lower frequencies transmission increases again to an asymptotic value, which increases to 1 for low temperature atmospheres (perfect transmission). Two main factors produce this behavior at low frequencies. The first is due to Alfvén speed gradients which are stronger in the low atmosphere, vanish close to the base, and then increase slightly before going asymptotically to zero. This profile produces a tunneling effect at low frequencies and it accounts for the increase of $T$ for decreasing frequencies in the range $5 \times 10^{-6}$ Hz $\lesssim \omega \lesssim 5 \times 10^{-4}$ Hz (a feature found also in static atmospheres). The second factor is the presence of a wind, which alters the propagation of waves and carries the low frequency modes ($\omega \gtrsim 5 \times 10^{-6}$Hz) through the critical point, enhancing the transmission. Therefore, an initial frequency spectrum of Alfvén waves modifies its shape during its propagation in the inner corona, without the development of nonlinear interactions. In term of the cross helicity this variation results in a decreasing $\sigma_c$ at low frequencies and an increasing $\sigma_c$ at high frequencies. The two regimes (static fluctuations and WKB fluctuations respectively) are separated by a critical frequency which is independent of distance in the limit of uniform wind speed and radial expansion (a condition satisfied in the supersonic solar wind) but the radial dependence of $\sigma_c$, in contrast to observations which show the opposite behavior (Velli et al. 1991). The linear analysis hence predict a variation of the frequency spectrum which is opposite to what is observed: a possible solution to this discrepancy may come from the inclusion of nonlinear terms in the equations.
Nonlinear Interactions: the Effect of Asymmetries in the $\epsilon^\pm$ Energies and of The Wind Expansion

As mentioned above, solar wind turbulence is more complex than the homogeneous picture presented in the phenomenological discussion of MHD turbulence (sec. 2.3.1). In the Alfvénic range the energy residing in the outward propagating fluctuations is larger than that in the inward propagating ones and expansion effects must be taken into account for the radial evolution of these two modes. The imbalance in the fields can be retained in the phenomenological derivation of the energy fluxes. Assuming coherent nonlinear interactions (as in the Kolmogorov case), one obtains two distinct fluxes in the wavenumbers, $\Pi^\pm_k$, for both the species (Mangeney et al. 1991):

$$\Pi^\pm_k = k^{5/2} \frac{E^+_k E^-_k}{(E^+_k)^{1/2}} = \epsilon^\pm,$$

(2.39)

where $\epsilon^\pm$ are the energy fluxes in the inertial range and $E^\pm_k$ are the Elsässer energies at wavenumber $k$ in the two modes. Assuming constant energy fluxes leads to a Kolmogorov like spectrum (slope -5/3) for both modes independently of the asymmetry in $\epsilon^\pm$. Still retaining isotropy but allowing for the decorrelation effect (Kraichnan phenomenology) one obtains instead:

$$\Pi^+_k = \Pi^-_k = k^3 \frac{E^+_k E^-_k}{V_a} = \epsilon^\pm,$$

(2.40)

which reduces to the IK spectrum for fluctuations of comparable amplitudes (slope -3/2). Anisotropies in the cascade modify the above expression as,

$$\Pi^+_k = \Pi^-_k = k^4 \frac{E^+_k E^-_k}{k_0 V_a} = \epsilon^\pm.$$

(2.41)

In both cases (eqs. 2.40-2.41) the energy flux is the same for both modes, whatever the imbalance, but no defined scaling law is recovered for the energy spectrum (or additional physics of assumptions must be used). One can only say that the sum of the slope of the two spectra $E^\pm_k \propto k^{m_\pm}$ must satisfy $m_+ + m_- = 3$, 4 for the isotropic and anisotropic IK case respectively.

Simulations for isotropic homogeneous MHD turbulence (Grappin et al. 1982; Matthaeus et al. 1983; Grappin 1986; Pouquet et al. 1986) showed that an initial imbalance between the two modes evolves in a way to enhance the initial asymmetry, so that the magnetic and velocity field are more and more aligned (dynamic alignment, Dobrowolny et al. 1980). Since the dissipation of both mode proceeds at the same rate (see eq. 2.40), the cross helicity $C = |2\mathbf{u} \cdot \mathbf{b}| = |E^+ - E^-|$ remains constant while the total energy $E = E^+ + E^-$ decays in time. As a result the normalized cross helicity $C/E$ increases in time driven by the turbulent dynamics. Actually, from numerical simulation, it has been found that the cross helicity decreases with time at a slower rate than the energy, so the net result is the same as above.
The inclusion of wind expansion to explain the evolution of the break observed in the frequency spectra was first attempted by Tu et al. (1984). Fourier transform of eq. 2.26, in a stationary state, yields the equation describing the evolution of the spectra, \( E^\pm \), which depend both on wavenumber \( k \) and in the slow variable \( r \):

\[
\nabla \cdot (V_g^\pm E_k^\pm (r)) + E_k^\pm \frac{1}{2} \nabla \cdot U + M_k^\pm (r) = \frac{\partial \Pi_k^\pm}{\partial k}.
\]

(2.42)

On the LHS derivatives are taken with respect to the slow variable \( r \), the first term is the energy flux, the second the work done by the wave on the wind, while the third term represents the usual linear coupling given by the large scale inhomogeneities,

\[
M_k^\pm (k, r) = -z_{k^\pm} \frac{1}{z_{k^\pm}} \left[ \nabla V_g^\pm - \frac{1}{2} \delta / \nabla \cdot \left( V_a \pm \frac{1}{2} U \right) \right].
\]

(2.43)

which couples the \( E^\pm \) equations to that of the energy difference \( u^2 - b^2 = z^+ \cdot z^- \). Tu et al. (1984) and Tu (1988) solved the above eq. 2.43 with a phenomenological expression for the energy flux similar to eq. 2.39-2.40 (hence isotropy is assumed), in which the ratio \( \alpha = E^+ / E^- \) is constant, fitting an average observed value. They retained only the WKB term in the evolution equation \( (M^\pm = 0) \), written for a spherically symmetric background wind. At low frequencies the nonlinear timescale is much greater than the adiabatic timescale \( (\tau_{nl} >> \tau_{ad} \approx R / U) \) while at high frequencies the opposite inequality holds. The scale dividing the two regimes, \( L(r) \), evolves with distance (expansion effect) modifying both the timescales. For \( k < 1 / L \) the WKB effects determine the evolution of the spectrum, which decays self-similarly as \( 1 / r \) since nonlinear effects are negligible; for \( k > 1 / L \) the nonlinear interactions dominate and an equilibrium spectrum is maintained \( (k^{-5/3} \text{ or } k^{-3/2} \text{ depending on the energy flux chosen}) \). Starting with a flat spectrum \( (k^{-1}) \) at 0.3 AU similar to that given by observations, they were able to reproduce the steepening of the frequency spectrum, first at high frequencies and later on at small frequencies, with a break in the spectrum at a scale \( 1 / L(r) \) which evolves consistently with observations \( (L \text{ increases with distance, increasing the domain in which the spectrum approaches the equilibrium spectrum}) \).

With a similar approach, Velli et al. (1989, 1990) tried to find an explanation for the flat spectrum which was assumed to be already formed at the lower boundary of the above model (and naturally leading to its evolution, being different from the asymptotic equilibrium solution). They showed that starting with a purely outward propagating wave, at the lowest order of the WKB approximation, for low frequency waves (those satisfying \( \tau_{nl} |\nabla \cdot U| < 1 \), i.e. at 0.3 AU for period smaller than about 2 h) there is a reflected component generated by the inhomogeneity, with the same phase of the outward propagating mother wave. This anomalous reflected component triggers nonlinear interactions and the resulting spectrum (mainly given by the outward
2.3. Turbulence in the Fast Solar Wind

spectrum, $z^- \ll z^+$) as a $k^{-1}$ slope, while the reflected component displays a steeper spectrum ($k^{-3}$). This could explain the $k^{-1}$ spectrum as an outcome of the anomalous cascade for fluctuations coming from the Sun, in absence of the local production of the “regular” inward component, as generated by shear interaction for example, which would otherwise dominate (leading to the asymptotic equilibrium). The problem with this scenario is that the time to establish such spectrum is too large when $z^-$ is too low (see comments by Zhou et al. 1990 and the authors reply, as well as, Velli et al. 1990).

2.3.4 Our Approach to Alfvénic Turbulence

The anisotropy of fluctuation, the imbalance in the outward and inward modes and the effect of expansion make the solar wind turbulence very different from the idealized homogenous isotropic turbulence usually investigated in MHD simulations. Resolve all these effects at once is a task beyond current capability and one has to investigate the above issue in some simplified framework. Our understanding of the flat spectrum found at 0.3 AU is far from satisfactory, neither the contribution of turbulence to the solar wind heating and acceleration, nor how the turbulent cascade develops in anisotropic and expanding geometry is understood as well. To address these issues, we consider anisotropic fluctuation propagating parallel to the mean magnetic field in the expanding solar wind, from the photosphere to the heliosphere, allowing inhomogeneity and nonlinear interactions. The linear coupling due to the density gradients is taken into account self consistently (no ad hoc closure) together with nonlinearities, whose effect is modeled either phenomenologically or through a low-dimensional model for turbulence (shell models). The nonlinear coupling occurs on planes perpendicular to the direction of propagation, assumed to be parallel to the mean magnetic field. No parallel spectral transfer occurs, rather, the modification of the parallel wavenumber spectrum is due to the inhomogeneities (reflection effects) which, triggering the nonlinear interactions and hence the cascade in the perpendicular wavenumber, modify the energy content of the fluctuations with respect to the linear behavior. Therefore the frequency spectra that are obtained are the result of the modification of the linear evolution of outward propagating fluctuations induced by the self generated turbulent dissipation; turbulence does not develop everywhere but it is the outcome of the reflection mechanism discussed in sec. 2.3.3. With the phenomenological model, the large scale properties of turbulence are tested in relation to the problem of acceleration and heating of the solar wind. With the low-dimensional model for turbulence, we are able to give more detailed estimations on the possible onset of turbulence and its resulting properties, with the drawback of completely neglecting the reaction of the wind to the turbulent heating.
2. The Fast Solar Wind, Heating and Acceleration Mechanisms
Chapter 3

A Semi-Analytical Approach

A linear analysis applied to eq. 2.26 shows that reflection depends strongly on the wave frequency (Velli 1993) as discussed in paragraph 2.3.3. Low frequency waves are the most reflected and hence they are supposed to be the driving modes for nonlinear interactions. In this chapter we consider a simplified model in which nonlinearities and dissipation are taken into account adopting a phenomenological term in favor of a rigorous treatment of the propagation and reflection properties of the waves. In order to construct the model equations the following steps are required:

1. Linearize eq. 2.26, neglecting hence all the terms in the RHS.
2. Steadiness of the ambient fields \((U, B, \rho)\) allows to fourier transform the Elsässer variables, we use then:
   \[
   z^\pm(r, t) = z^\pm(r) \exp[-i\omega t] \tag{3.1}
   \]
3. Retains explicitly only the radial dependence of the variables, i.e.
   \[
   U = U(r)\hat{e}_r, \quad V_a = V_a(r)\hat{e}_r, \quad z^\pm = z^\pm(r)\hat{e}_\perp \tag{3.2}
   \]
4. Provide a model for the phenomenological nonlinear term and add it to the linearized, 1D equations obtained.
5. Provide a model for the ambient fields in order to have specified all the coefficients in the equation for the Elsässer fields.

The form of the nonlinear dissipative term is chosen to preserve the main features of the Alfvén waves nonlinear interaction and turbulent dissipation: it involves only counter-propagating waves and contains a characteristic length scale \(\lambda\) which is the similarity length scale of single point closure model for
turbulence (Dobrowolny et al. 1980; Grappin et al. 1982). The general form of the phenomenological term is then:

\[ \mathcal{N}_L = -z^\perp \cdot \nabla z^\perp \approx -\frac{Z^\perp}{2\lambda} z^\perp = -nl^\perp z^\perp. \]  

(3.3)

Basically the replacement \( z^\perp \cdot \nabla \rightarrow \frac{Z^\perp}{2\lambda} \) hides the whole dynamics in the perpendicular planes which is actually fundamental for the development of the turbulent cascade. Beside the simple replacement the form chosen can be justified with an heuristic argumentation (following the one given by Dobrowolny et al. 1980). When eq. 2.26 is Fourier decomposed \((z^\perp(r, t) \rightarrow z^\perp_k = u_k \mp b_k)\) nonlinear terms couple several wavenumbers in the \( k \)-space. When a strong magnetic field \((V_a \text{ in velocity units})\) is present, the propagation time of the Alfvén waves \( \tau_a = (k \cdot V_a)^{-1} \) is equal or shorter than the characteristic timescale for nonlinear interaction \( \tau_{NL} = (k u_k)^{-1} \approx (k b_k)^{-1} \) over most of the Fourier space, the nature of the nonlinear cascade is highly anisotropic, developing preferentially in planes perpendicular to the direction of the mean field (Shebalin et al. 1983; Oughton et al. 1994; Goldreich & Sridhar 1995). It is then useful to decompose local wavenumber in projections along the magnetic field \((k||)\) and in the perpendicular planes \((k\perp)\) because energy transfer occurs only among the latter, so that Fourier decomposition is exploited only in \( k\perp \). When small fluctuations are considered \((V_a \gg b_k \approx u_k)\) these arguments lead to the so called RMHD description which can be derived as an expansion of the usual MHD equations in the small parameter \( b_k/V_a \) with the restriction \( \epsilon_{RMHD} = \tau_{NL}/\tau_a \lesssim 1 \) (see Oughton et al. 2004 and reference therein for more details on RMHD), in which variations along the perpendicular directions are decoupled from those along the magnetic field \((\nabla = \nabla_\perp + \nabla_{||} \text{ with } \nabla_\perp \gg \nabla_{||})\). We can describe the global effect of this perpendicular cascade by means of two quantities at the large scales, namely an integral scale \( \lambda_0 \), giving the dimension of the greatest eddies in which energy is injected, and the average velocity difference \((\Delta v)\) among points belonging to the same eddy, which in RMHD turbulence also contains magnetic field fluctuations in velocity units \((\Delta b/\sqrt{4\pi \rho})\). Identifying these two quantities with the integral turbulent length \((\lambda_0 = A)\) and the fluctuations’ amplitude of the Elsässer fields we can construct a characteristic timescale \( \tau_{NL} = A/[|Z^\perp|] \) which accounts for nonlinear turbulent interactions in eq. 2.26. The factor of two in the denominator in inserted to have a dissipation rate in the energy equation consistent with that derived with 2D MHD simulations (Matthaeus et al. 1999; Hossain et al. 1995). If one in fact multiplies the general form of the equations,

\[
(U \pm V_a) \frac{dz^\pm}{dr} - i\omega z^\pm + \frac{1}{2A} \frac{dA}{dr} (U \mp V_a) z^\mp + \frac{1}{2} (z^- - z^+) \left[ \left( \frac{1}{A} \frac{dA}{dr} + \frac{d}{dr} \right) \left( V_a \mp \frac{1}{2} U \right) \right] = \mathcal{N}_L^\pm,
\]

(3.4)

by the complex conjugate of the Elsässer fields and adds the two equations one finds that the rate of change of the total energy per unit mass due to
dissipation is given by:

\[
\frac{dE_{\text{tot}}}{dr}_{\text{diss}} = \left( \frac{1}{2} \frac{dZ^2}{dr} \right)_{\text{diss}} + \left( \frac{1}{2} \frac{dZ^{-2}}{dr} \right)_{\text{diss}} = -\frac{Z^2Z^- + Z^{-2}Z^+}{2A} \quad (3.5)
\]

Once nonlinear terms are introduced we loose the scaling feature of the linearized equation for which given an increment of a factor \( f \) in the amplitude of \( z^+ \) at the critical point we have an equal increment \( f \) for the values at the base. Since the wave amplitude is an input parameter for the problem, one has to choose where this amplitude should be imposed. The regularity condition at the Alfvénic critical point (\( X_a \), cfr. section 2.3.3) suggests to use \( X_a \) for a parameter space study. When applied to a specific case, such as the solar wind, one prefers to impose the amplitude at the base of atmosphere as a physical boundary condition. To get the desired values we have to tune the amplitude of the outgoing wave imposed at the Alfvénic critical point with an iteration procedure, integrating backward to the base for each form of the NL term considered.

Note that adding nonlinearities in the equation yields a modified version of eq. 2.28 which is still independent of the phase. The coefficient \( \mu \) used to derive the boundary conditions at \( X_a \) becomes (cfr. eq. 2.34):

\[
\mu_{\text{NL}} = \frac{1}{2} \left( \frac{dU}{dr} + \frac{1}{A} \frac{dA}{dr} U + nI^- \right)_{r=X_a} \quad (3.6)
\]

It follows that for high enough \( z^+_a \) the energy density spectrum is flat (no frequency dependence) while the profile tends to the decreasing spectrum found in the linear case for small value of \( z^+_a \).

In the following section we investigate the effect of different models for nonlinearities and turbulent dissipation in atmospheres permeated by an isothermal wind, of different temperatures, varying the wave amplitude as well. The next section is an application to the solar atmosphere for which a non-isothermal wind is considered and in which the photosphere and the chromosphere are also considered together with the transition region, or in other words we introduce layers in which reflection is so strong to overcome the nonlinear coupling. As a final application of this model we discard the first layers and focus only to the solar corona in order to consider the back reaction of the waves (heating and work spent in accelerating the wind) on the mean flow. This last section includes also an energy equation for the mean flow quantities and use a time dependent code to solve the (M)HD equations. The non-dimensional wave equations are obtained giving a normalization value for length, speed and time and using typical coronal base values for the quantities defining the atmosphere. The chosen normalization constants, except when explicitly given, are:

- Temperature in units of \( T^* = 10^6 \) K
• Numerical density in units of \( n^* = 10^8 \text{ cm}^{-3} \)

• Magnetic field in units of \( B^* = 1 \text{ G} \)

• Length in units of \( R^* = R_{\odot} \)

• Speed in units of \( u^* = 128 \text{ km s}^{-1} \), corresponding to the sound speed in a fully ionized hydrogen plasma at \( T = 10^6 \text{ K} \)

• Time in units of \( t^* = R^*/u^* \) and frequency in units of \( 1/t^* \)

The equations defining the atmosphere and the waves are integrated numerically with an ODE solver with adaptive step size, the Bulish-Stoer extrapolation method (Press et al. 1986). For non isothermal atmosphere with wind, as in sec. 3.2, the sonic critical point is not known a priori and an iterative procedure is used to find the wind transonic solution (see Casalbuoni et al. 1999 for details).
We study the nonlinear evolution of Alfvén waves in a radially stratified isothermal atmosphere with wind, from the atmospheric base out to the Alfvénic point, varying the temperature of the isothermal wind (which controls reflection), the wave amplitude and the frequencies involved in the nonlinear coupling. Nonlinear interactions, triggered by wave reflection due to the atmospheric gradients, are assumed to occur mainly in directions perpendicular to the mean radial magnetic field. The nonlinear coupling between waves propagating in opposite directions is modeled by a phenomenological term, containing an integral turbulent length scale, which acts as a dissipative coefficient for waves of a given frequency. Although the wind acceleration profile is not determined self-consistently one may estimate the dissipation rate inside the layer and follow the evolution of an initial frequency spectrum. Reflection of low frequency waves drives dissipation across the whole spectrum, and steeper gradients, i.e. lower coronal temperatures, enhance the dissipation rate. Moreover, when reasonable wave amplitudes are considered, waves of all frequencies damp at the same rate and the spectrum is not modified substantially during propagation. Therefore the sub-Alfvénic coronal layer acts differently when waves interact nonlinearly, no longer behaving as a frequency dependent filter once reflection-generated nonlinear interactions are included, at least within the classes of models discussed here.

3.1.1 Model Atmosphere

The isothermal atmosphere is completely defined by setting the values for temperature, density and magnetic field intensity at the base, together with the mass and radius of the central object ($M_\odot$ and $R_\odot$). The wind speed and Alfvén speed profiles (and their derivatives) depend on the two parameters $\alpha$, the non-dimensional scale height, and $\beta_0$, the plasma parameter at the base,

$$\alpha = \frac{GM_\odot}{R_\odot c_s^2} \approx \frac{v_{esc}^2}{c_s^2} \quad \text{and} \quad \beta_0 = \left( \frac{P}{B^2 / 8\pi} \right)_0 \approx c_s^2 \frac{V_a^2}{V_{a0}^2},$$

where $c_s$ is the sound speed and $v_{esc}$ is the escape speed from the stellar surface. This allows one to solve numerically the implicit equation for the isothermal wind,

$$\frac{1}{2} \left( \frac{U}{c_s} \right)^2 - \log \left( \frac{U}{c_s} \right) = 2 \log \left( \frac{2 R}{\alpha R_\odot} \right) + \alpha \frac{R_\odot}{R} - \frac{3}{2},$$

(3.7)
from which the profile for Alfvén speed (as usual $R$ is the heliocentric distance, here $\gamma_g = c_p/c_v$, while $r = R/R_\odot$) follows:

$$V_a(r) = \frac{1}{r} \sqrt{\frac{2}{\gamma_g \beta_0} \frac{U(r)}{U_0}} \approx \beta^{-1/2}(r).$$ (3.8)

The values at the base for mass density and magnetic field intensity are related by the Alfvén speed definition ($\rho_0 = B_0^2/4\pi V_a^2$), so one has to impose only one of the two, while their profiles are fixed by flux conservation equations,

$$\rho(r) = \frac{\rho_0}{r^2} \frac{U_0}{U(r)}, \quad B(r) = \frac{B_0}{r^2}.$$

### 3.1.2 Nonlinear model

Since nonlinear interactions involve waves of different frequency it is desirable to include in the phenomenological term a kind of non-local coupling in the frequency space. A way to do it consists in considering nonlinear interactions with the frequency integrated counter-propagating Elsässer variable (or with its root mean square value) instead of with its single frequency fourier component (see fig. 3.1 for a representative scheme). The interactions between waves of the same frequency do not match the resonant condition required for the development of a turbulent cascade unless their frequency is so slow so that the nonlinear timescale is much smaller than the Alfvén timescale (cfr. section 2.3). However this class of interactions represents the zero-order approximation when including nonlinearities in the equations, and also allows a direct comparison with linear results. Therefore it will be the first case study,
while later on more frequencies will be included in the nonlinear model. The general form of the nonlinear term is

\[ \mathbf{N} L \mathbf{F} \times (\omega_j) = \frac{z^j(\omega_j) \sqrt{\sum_i^n |z^i(\omega)|^2}}{2 \lambda}. \] (3.9)

The single frequency interaction clearly implies \( i = j \). As boundary condition we assign a frequency spectrum for the outgoing mode at the Alfvénic critical point, i.e. \( z^a_0(\omega) \).

### 3.1.3 Results

To quantify the effect of nonlinearities inside the layer is better to introduce some ad hoc quantities whose definition relies on the conservation properties of the linear equations. The quantity

\[ S(r) = \frac{S^*(r)}{S^*(r_0)} \] (3.10)

highlights the variation of the wave action density with \( r \) compared to the linear case (for which \( S(r) = 1 \)). Since the extent of the layer depends on the temperature selected to quantify the integrated effect of dissipation, we use the “dissipation efficiency”:

\[ \gamma = \frac{S^*(r_0) - S^*(X_a)}{S^*(r_0)} = 1 - S(X_a). \] (3.11)

The above quantities do not directly yield the amount of energy dissipated, however, the profile of \( S(r) \) shows where the nonlinear dissipative terms have the strongest effect.

Further interesting information comes from studying how the values of the Elsässer variables at the base depend on the value at the critical point, departure from a linear scaling being entirely due to nonlinear effects. For solar coronal temperature these amplitudes are constrained by measures of line broadening which give root mean squared values of the velocity field fluctuations approximately between \( 20 \text{ km s}^{-1} \) and \( 30 \text{ km s}^{-1} \) (Chae et al. 1998).

In the following we first present the results concerning the self-interacting case, where only monochromatic waves interact, then we shall consider modification to dissipation induced by different coupling among two or three frequencies.

\( ^1 \)A flat spectrum (same outgoing wave amplitude) implies a rms amplitude \( Z^* = \sqrt{N}\sigma_0^2 \), where \( N \) is the number of frequencies. The frequency coupling in this section will involve at best three waves so that comparison between single frequency and multi frequency model in which the same wave amplitude is imposed can be done safely. For more than 3 frequency coupling a different evaluation of \( Z \) will be given, as in section 3.2.
Figure 3.2: Total wave action density normalized to the base value as a function of radius in $\alpha = 4$ and $\alpha = 10$ atmospheres, for a high ($10^{-2}$ Hz, dotted line) and low ($10^{-6}$ Hz, solid line) frequency wave with an initial $100 \text{ km s}^{-1}$ wave amplitude. For the cold atmosphere the profile for the wave having the minimum transmission coefficient (dashed line) is also plotted.

As model parameters we choose $\alpha \in [4, 10]$ corresponding to temperatures for an isothermal layer above the Sun surface ranging from approximately $1 \times 10^6 \text{ K}$ ($\alpha = 10$) to $3 \times 10^6 \text{ K}$ ($\alpha = 4$), and we fix the value of the plasma parameter at $\beta_0 = 0.08$ for every temperature; thus, at the base of the atmosphere the Alfvén speed is always five times the sound speed. Finally we set the value of the phenomenological turbulent length $2\lambda = 0.05 R_\odot$, that is about 34000 km corresponding to the average size of the supergranule at coronal level which is maintained at a constant value through the entire atmosphere (except in the last section).

**Self Interacting Case**

Consider now nonlinear interactions which couple (only) counter-propagating waves of the same frequency, with the nonlinear terms ($NL$) having the following form:

$$NL \to -\frac{x^\pm(\omega)|z^\pm(\omega)|}{2\lambda}$$

Although this type of interaction is not dominant it is nonetheless a useful step in this kind of modelling in order to understand the behavior of the waves evolution, in particular it reveals the importance of the local reflection rate in determining the wave dissipation.

In fig. 3.2, $S(r)$ is displayed for a high and a low frequency wave ($\omega = 10^{-2}$ Hz and $\omega = 10^{-6}$ Hz respectively) traveling in $\alpha = 4$ and $\alpha = 10$ atmospheres, with a reference initial wave amplitude value of $100 \text{ km s}^{-1}$. For the high temperature case (left panel), one obtains what is expected from the transmission
3.1. Application to Solar-like Atmospheres Permeated by Isothermal Wind

Figure 3.3: Dissipation efficiency ($\gamma$) as a function of frequency for a hot ($\alpha = 4$, continuous lines) and a cold ($\alpha = 10$, dotted lines) corona for (a) $z_a^0 = 100$ km s$^{-1}$, (b) $z_a^0 = 10$ km s$^{-1}$ and (c) $z_a^0 = 1$ km s$^{-1}$.

Coefficient found in the linear analysis: high-frequency waves are poorly dissipated while at low frequencies dissipation is enhanced, eventually with more efficiency than one expects on the basis of the transmission coefficient ($\approx 80\%$). For low temperatures (right panel), one expects high dissipation for waves with periods of a few hours (transmission $\approx 20\%$) and that is what is found (dashed line), but for the lower frequency waves one finds the same amount of dissipation despite almost perfect linear transmission ($T \approx 95\%$). The reason is that when non-negligible amplitudes are considered, the nonlinear terms dominate over the linear ones where the gradients are strongest and the local reflection rate determines the amount of dissipation. In the right panel, at about $r = \alpha/4$, dissipation stops and total wave action density remains almost constant just because the reflection vanishes.

The local reflection rate depends both on temperature and wave frequency and it is higher for cooler atmospheres and lower frequency waves. Moreover, most of the reflection takes place in the lower atmosphere, where the Alfvén speed gradients are stronger. For high enough wave amplitudes, nonlinear terms dominate over linear ones and the profile of $|\mathbf{a}|$ is determined uniquely by the local reflection rate, which, for cold atmospheres, is itself dominated by the Alfvén speed gradients.

As a consequence one expects $\gamma$ ($\equiv 1 - S_\omega$) to increase with $\alpha$ (decreasing temperature), and $S$ to decrease faster with radius in the very low atmosphere. For a given $\alpha$ this behavior is more pronounced for low-frequency waves (those suffering stronger reflection) and consequently high-frequency dissipation should be less sensitive to temperature variations. What is still to be determined is the discriminant between high and low frequency.

Variation of dissipation efficiency with frequency is shown in fig. 3.3 for differ-
ent initial wave amplitudes (a → 100 km s$^{-1}$, b → 10 km s$^{-1}$ and c → 1 km s$^{-1}$) and both $\alpha = 4$ and $\alpha = 10$ atmospheres (respectively continuous and dotted lines). One can define a critical frequency $\omega^*$ (for which $\gamma$ is half its maximum) that divides the $\gamma(\omega)$ profiles into two branches: a constant low-frequency value and a zero high-frequency one, separated by an intermediate range whose extent depends slightly on temperature and wave amplitude (it decreases with $\alpha$ and $z^+_a$). As can be seen in fig. 3.3 the value of the critical frequency (marked with a star) depends very little on temperature while it seems to scale almost linearly with the value of the amplitude imposed at the top of the atmosphere, indicating that $|z^+_a|$ is now a more relevant parameter than temperature. The difference between dissipation efficiency for low and high frequency ($\Delta \gamma$) depends also on $\alpha$ and the initial wave amplitude and it obviously increases as either parameter is increased.

To investigate the amplitude dependence of the solutions we consider the case of a frequency small enough ($\omega = 10^{-6}$ Hz) to remain to the left of $\omega^*$ for all temperatures investigated. In fig. 3.4 the values of the outgoing and ingoing wave amplitudes at the base are plotted as a function of the outgoing wave amplitude imposed at the critical point (i.e. the energy). The value at the base for the outward propagating waves follows a profile similar to the linear case, it is increased by about the same factor as the initial value at the top of the layer, suggesting that dissipation mostly affects inward propagating waves. As a further indication, their amplitude first grows almost linearly (initial amplitude is not too high and dissipation acts like a perturbation to the linear problem). Then, as $z^+_a$ further increases, $z^-_p$ decreases and vanishes for a particular value ($z^+_p$) imposed at the critical point. Finally as $|z^+_a|$ still
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Figure 3.5: Dissipation efficiency as a function of initial outgoing wave amplitude for $\omega = 10^{-6}$ Hz. The different plots refer to atmospheres with $\alpha = 4, 6, 8, 10$.

grows, $z_0^-$ reaches a very low constant value (say $|z_0^-| \approx 10$ km s$^{-1}$), indicating a kind of saturation which limits inward wave amplitudes at the base. In fig. 3.5 dissipation efficiency is plotted as a function of initial wave amplitude for $\omega = 10^{-6}$ Hz and $\alpha$ equal to 4, 6, 8 and 10. For low amplitudes $\gamma$ is very low and for the highest two temperatures (negligible reflection) it has the same numerical value. Increasing $z_0^+$ we arrive to a saturation value depending on temperature (logarithmic scale on x axis) which corresponds to the saturation of the $z^-$ amplitude at the base of the atmosphere. In the presence of nonlinear interactions it is interesting to remark that the lack of an appreciable reflected wave at the base is not a sign of low dissipation, since as shown above, $z^-$ may be generated locally and dissipated entirely within the atmospheric layer.

Two Frequency Interactions

We consider here the effect of a frequency coupling in the nonlinear term. In the following we refer to $\omega_0$ as the basic frequency, or fundamental, with which interactions occurs, while with $\omega_i$ we refer to the “interacting” frequencies. Since in a decaying power law spectrum most of the energy is retained in the low frequencies we expect nonlinear interaction to be more important when such frequencies are involved, hence we choose $\omega_0 = 0$ (representative of low frequencies, say $\omega < 5 \times 10^{-5}$ Hz) and $\omega_i$ varying from about $10^{-5}$ Hz to $10^{-2}$ Hz increasing by a factor 10 at each step. Specifically, we consider the four couplings,

\[
a : \omega_0 = 0 \text{ Hz} - \omega_i = 10^{-5} \text{ Hz},
\]
\[
b : \omega_0 = 0 \text{ Hz} - \omega_i = 10^{-4} \text{ Hz},
\]
\[
c : \omega_0 = 0 \text{ Hz} - \omega_i = 10^{-3} \text{ Hz},
\]
Figure 3.6: Flat spectrum. *Left panel.* Total wave action density, normalized to the base value, for the interacting frequencies in different coupling (labelled with letters, see text), as a function of distance from the atmosphere’s base expressed in unit of $R_\odot$. Temperature is set to $\alpha = 6$ and $z_0^\alpha(\omega_{ref}) = 100$ km s$^{-1}$. *Right panel.* Dissipation efficiency as a function of initial outgoing wave amplitude $z_0^\alpha(\omega_{ref})$. The different plots refer to atmospheres with $\alpha = 4, 6, 8, 10$; solid and dotted lines represent respectively $\gamma(\omega_0)$ and $\gamma(\omega_i)$ with $\omega_i = 10^{-2}$ Hz.

Figure 3.7: Same as fig. 3.6 but for a power-law spectrum at the critical point.

d : $\omega_0 = 0$ Hz - $\omega_i = 10^{-2}$ Hz.

As top boundary conditions two cases are considered: a flat spectrum (same energies in the fundamental and interacting modes) and a Kolmogorov-like spectrum, for which the energy per unit mass scales as $|z_0^\alpha(\omega)|^2 = |z_0^\alpha(\omega_{ref})|^2 \times (\omega/\omega_{ref})^{-2/3}$, where $\omega_{ref} = 10^{-6}$ Hz is a reference frequency from where the power-law scaling begins ($\omega_{ref} \equiv \omega_0$).
Consider first the flat spectrum case shown in fig. 3.6 (left panel). The general dissipation profile as a function of $r$ is similar to that formed in fig. 3.2 (right panel). The main result here is that higher frequency waves ($b, c, d$) may also dissipate efficiently thanks to their coupling with the reflected mode of the very low frequency component, so that all profiles show a significant decrease with distance. This also has an effect on the decay of the lowest (zero) frequency mode, whose reflected mode is ultimately the trigger for nonlinear evolution. When the amplitudes of the higher frequency modes are large (i.e. of the same order of magnitude of the low frequency mode), they influence the evolution of the zero frequency reflected component, which then also affects the zero-frequency outward component, driving the profiles to convergence as illustrated in fig. 3.6 (right panel): at low amplitudes $\gamma(\omega_i)$ differs from $\gamma(\omega_0)$ but at high amplitudes $\gamma(\omega_i) \approx \gamma(\omega_0)$, which means a strong coupling.

Consider now the power-law spectrum illustrated in fig. 3.7. The dissipation profile is the same for all the couplings considered (left panel). Now the energy in the high-frequency waves (mainly propagating outward) is so small that it has a negligible effect on the evolution of the zero-frequency mode (no back reaction) which, in turn, drives the dissipation of all the coupled modes. On the other hand, in the right panel, one can see that the profiles of $\gamma(\omega_0)$ and $\gamma(\omega_i)$ are similar to the flat spectrum case and they begin to converge at almost the same values of $z^* + a(\omega_0)$.

The two parameters, $\alpha$ and $z^* + a(\omega_0)$, determine how strong the coupling is, independently of the shape of the spectrum, in other words if the fundamental and interacting modes have the same dissipation profile of $S(r)$. For a given amplitude at the critical point of the zero-frequency wave, temperature controls the amount of reflection produced inside the layer, and hence both the linear coupling among the counter-propagating waves of a given frequency (i.e. differences in the waves’ evolution due to frequency differences) and the amplitude of the zero-frequency reflected component (the driver). One finds that increasing the temperature (decreasing $\alpha$) the coupling becomes weaker for a low $|z^*| \text{ and stronger for a high } |z^*|$, depending on which of the two above features is dominant. For a given coupling, $\omega_0 - \omega_i$, at a given temperature, the zero frequency wave amplitude imposed at the critical point determines the importance of nonlinear terms (see fig. 3.4), and hence the nonlinear coupling among the waves (the evolution independent of frequencies). Generally, increasing $|z^*(\omega_0)|$ increases the strength of the coupling.

Three Frequency Interactions

Including a third frequency in the nonlinear terms represents not only a simple improvement of the previous two interacting frequencies model but it allows one also to have a rough guide for the evolution of the initial spectrum due to wave propagation. Since the strength of coupling depends both on the temperature of the layer and on the zero-frequency wave amplitude, one expects that
modifications of the initial spectrum will be non-negligible only for very low temperature (high $\alpha$) or very low initial amplitude. If in fact dissipation efficiency is the same for all the frequencies coupled the spectrum should remain almost unchanged during wave propagation. We shall consider only the power-law case since it better represents the condition of a fully developed turbulent spectrum and the strength of the coupling is not sensitive to the flatness of the spectrum imposed at the top boundary. The energy distribution among frequencies can be imposed by simply specifying the amplitude of the outgoing wave at a given frequency (cfr. eq. 3.6).

Supported by these arguments we shall study the behavior of efficiency and spectra with respect to “initial” wave amplitude variation of two representative couplings:

\textbf{a} : $\omega_0 = 0$ Hz - $\omega_1 = 10^{-2}$ Hz - $\omega_2 = 10^{-1}$ Hz which involves high-frequency waves, \\
\textbf{b} : $\omega_0 = 0$ Hz - $\omega_1 = 10^{-4}$ Hz - $\omega_2 = 10^{-2}$ Hz which involves intermediate frequency waves lying in the domain of the correlations observed at 1 AU.

In the high frequencies coupling (a) the interacting dissipation efficiencies ($\gamma(\omega_1)$, $\gamma(\omega_2)$) in the left panel of fig. 3.8 practically coincide for all the initial amplitudes considered, while the differences with the fundamental one show again the previously identified dependences: for $\alpha = 4$ (and 6) high initial amplitudes (roughly greater than 50 km s$^{-1}$) produce strong couplings among the fundamental and interacting modes. However, when we consider cooler
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atmospheres, only the very high amplitudes (around 1000 km s\(^{-1}\)) are able to equalize the dissipation of all the modes.

For the intermediate frequency coupling (right panel), the three curves for the coupled waves follow different profiles and give evidence of how much coupling strength is frequency dependent: for the “middle” frequency (dotted line) dissipation efficiency soon reaches the fundamental mode regime even in the coolest atmospheres, while the highest frequency mode (dashed line) follows almost the same profile as in coupling \(a\).

It is then interesting to track the modifications of the imposed (at \(r_a\)) spectra back to the base of the atmosphere, for varying “initial” wave amplitudes. Only coupling \(b\) will be considered, for which one expects greater modifications. Fig. 3.9 plots the spectra \(\epsilon = \rho \left( |z^+(\omega)|^2 + |z^-(\omega)|^2 \right)\) imposed at the top of the atmosphere (dotted lines) and the spectra obtained by integration to the base of the atmosphere (solid lines). Results are shown for two different temperatures (\(\alpha = 4, 10\)). Three representative top-boundary wave amplitudes of the fundamental frequency are considered whose values are \(z_0^+(\omega_0) = 1, 100, 1000\) km s\(^{-1}\).

Since we impose a power-law spectrum at the top of the layer all dotted lines have slope \(-5/3\) and can be used as reference to see the modification induced by wave propagation. It is striking how much the spectra remain unchanged for practically all the temperature and all the initial amplitude considered and also if couplings with highest frequency waves are formed (not shown here), even though with a more accurate inspection one actually finds that the spectra change slightly (see for example solid the line marked with diamonds in the right panel).
A Semi-Analytical Approach

Figure 3.10: Same as in fig. 3.8 for coupling b and power-law initial spectrum except that integrations are performed using a spherically expanding turbulent length $\lambda(r) = \lambda_0 r$ with $\lambda_0$ fixed by the average dimension of supergranule at coronal level. Results are shown for $\alpha = 4, 10$ atmospheres.

The Effect of a Radially Expanding Turbulent Scale

Up to now we have considered a fixed turbulence scale, $\lambda$, in a wind which is spherically expanding. This assumption produces strong consequences on the dissipation rate in the higher part of the atmosphere and hence on dissipation efficiency (Zank et al. 1996; Matthaeus et al. 1998). In that region, in fact, Alfvén speed and wind speed gradients are weaker and we expect only a small amount of reflection. However, if the turbulence scale is held constant, the phenomenological gradient ($\sim 1/\lambda$) becomes more and more important as we move into the outer atmosphere since it is not expanding as the other gradients or length scales in eq. 3.4 are. Hence the nonlinear term increases and the net result is an enhancement of dissipation, especially for lower temperature atmospheres which extend up to about $40 R_\odot$. In order to correct such effects and to consider a consistent spherical expansion, we modify the three frequencies model to use $\lambda = \lambda_0 r$, with $\lambda_0$ fixed at the coronal level by the average dimension of the supergranule.

For low $z_c^*(\omega_0)$, dissipation efficiency is considerably altered (reduced) with respect to the non-expanding case (compare fig. 3.10 and fig. 3.8). Since nonlinear term influences are now reduced, we need greater initial amplitudes to efficiently couple the frequencies considered; as a consequence the profiles of $\gamma$ are shifted to the right with respect to the non-expanding model (by a factor of 3 or 4). For coupling a, the plots of $\gamma$ show the same kind of variations as discussed above and the profile of the interacting frequencies ($\omega_1$ and $\omega_2$) again coincides for every amplitude and temperature. Their values are identical to the high frequency one in coupling b (dashed lines in fig. 3.10). Now for low temperature and low amplitudes differences in the slopes are more evident
suggested some spectral evolution in such conditions.

### 3.1.4 Conclusions

As might be expected, lower temperature atmospheres, with higher gradients, and lower frequency waves allow a stronger dissipation of outwardly propagating waves. On the other hand, the results seem to imply that a well-developed turbulent spectrum does not change appreciably during propagation.

The dissipation rate has been studied, varying the temperature of the layer and the frequency and amplitude (imposed at the critical point) of the waves. We find that for a given amplitude and frequency, the dissipation rate is stronger in the lower part of the atmosphere and depends strongly on the temperature, which ultimately determines the amount of reflection via the density gradients. As the wave amplitude is increased, the dissipation rate is enhanced, reaching a saturation value which depends on temperature and frequency. Saturation is an effect of nonlinear-dissipative interactions which limit the inward propagating (reflected) wave amplitude once the outgoing amplitude is increased beyond a given value (which also depends on temperature and frequency). Below a critical frequency (whose temperature dependence is negligible), dissipation assumes a constant rate all the way down to “zero-frequency” fluctuations, while above the critical frequency the dissipation rate tends to zero as frequency is increased, since the amount of reflection decreases with frequency (WKB behavior). The local interaction analysis shows that even if the amplitudes of the inward propagating waves at the base are negligible, continuous reflection due to wind and Alfvén speed gradients can produce significant dissipation (see comments on fig. 3.5 at the end of paragraph 3.1.3); however, only low frequency waves are efficiently dissipated for reasonable wave amplitudes at the base of the corona.

We have also investigated a more realistic calculation, considering non-local interactions in the frequency space, for which an outwardly propagating Alfvén wave is allowed to interact nonlinearly with the total rms value of inward fluctuations summed over all frequencies (and vice versa). In this analysis, the description of the total rms value of the fluctuation is approximated by two or three representative waves with different frequencies spread across a spectrum, the “zero frequency” wave for the low frequencies, and the higher frequencies selected from spectra with different slopes. The strength of the coupling, i.e., the total energy in the outward modes at the lower frequencies (for which reflection is efficient) is crucial for the way in which energy is dissipated along the spectrum. If the amplitude of the low frequency wave is high enough (say $10 \text{ km s}^{-1}$ at the base for a $10^6 \text{ K atmosphere}$) dissipation of all the outward modes is driven by the low-frequency (quasi-2D) reflected waves: the coupling may be considered strong. A second
important aspect is the slope of the spectrum or equivalently the relevant energy residing in the higher frequency waves. Its effect is to enable dissipation of the low-frequency reflected component, since there is little reflected energy at high frequencies, and ultimately to reduce the dissipation efficiency of all the waves coupled. In summary, for a given total outward energy, dissipation is more efficient if the spectra have higher energies at lower frequencies, i.e. steeper spectral slopes.

Setting the lowest frequency to higher values, say $\omega_0 = 10^{-5}$ Hz, produces some differences in the results, but the global analysis remains unchanged. From fig. 3.3 one can guess how the strength of the coupling is affected and hence how the spectra change. In fact the higher the frequency is, the lower the amount of reflected waves; hence the lowest frequency wave is less efficient in driving the dissipation of all the waves coupled. Imposing the same amplitude at the critical point the strength of the coupling is lower, linear effects (i.e. differences in the wave propagation due to different reflection rates) become more important and the spectra show a somewhat higher modification. However, increasing the amplitude of the lowest frequency wave restores the importance of nonlinear terms (the coupling) which overcome the linear effects. These considerations remain valid if one chooses the lowest frequency in the low-frequency plateau of the curves in fig. 3.3. Note that the rightward extension of the plateau increases with increasing wave amplitude.

We have also considered a spherically expanding length scale $\lambda(r) = \lambda_0 r$. In this case, the qualitative features discussed above remain essentially the same (for high enough initial amplitudes). The main effect of the expansion is to reduce the phenomenological gradient $1/\lambda(r)$ entering the nonlinear term of eq. 3.4 as we move further out in the atmosphere so that dissipation in the higher part of the layer is greatly reduced. As a result the amount of energy dissipated is decreased and one can actually attribute differences in the dissipation rate of waves at different frequencies almost entirely to differences in their propagation through the low atmosphere where gradients are greater and hence where most of reflection takes place.

Of course, to better understand the spectral evolution in a stratified atmosphere one should also include the chromosphere, the photosphere and the transition region. Here different physical conditions are encountered and it is not clear how the evolution of the waves in these regions affects the development of “Alfvénic turbulence”.

Modeling the deeper stratified layers as a set of isothermal layer with different temperatures (with a discontinuity across the transition region) produces changes of the parameter $\beta$ which has been held fixed in this analysis. In our case, how changes in $\beta$ affect the result may be discussed qualitatively. First note that for a given temperature, the wind solutions are selected (see eq. 3.7) and hence the wind profile and its gradients remain the same. As $\beta$ is in-
creased the Alfvén speed decreases (still maintaining its characteristic profile, i.e. a maximum at $\alpha/4$), its gradients decrease too and the Alfvén critical point moves to lower radii. In the linear case the net effect is an enhanced transmission at low and intermediate frequencies. When nonlinear interactions are taken into account the amplitudes must be scaled to get the same quantitative results, i.e. the same amplitude may be regarded as low in a high $\beta$ plasma or high in a low $\beta$ plasma as far as the strength of the coupling is concerned because, for a given temperature, the strength depends entirely on the amount of reflection. When multiple isothermal layers with different temperatures and different thicknesses are considered, other characteristic length scales and gradients are introduced in the equation describing the wave propagation. The transmission properties of the entire atmosphere are altered, as are the properties of wave dissipation and the spectral slope.

The present results concern the evolution of a spectrum formed with only three frequencies which might not capture the whole shape modification in a complex atmosphere. In the model atmosphere, even if highly stratified, the density gradients, change gradually without discontinuities. However, the Alfvén waves observed in the solar wind do not necessarily originate at a photospheric level and can be generated directly in the corona. Moreover coronal structures can produce localized gradients so that the propagation of the waves can be dramatically altered with respect to the simple case studied here. One can then say that in an almost isothermal corona many processes, different from a turbulent cascade, are able to transform a complex signal into a simple spectrum as observed in situ in the solar wind.
3.2 Application to the Sun Atmosphere and Solar Wind

We solve the problem of propagation and dissipation of Alfvénic turbulence in a model solar atmosphere consisting of a static photosphere and chromosphere, transition region, and open corona and solar wind, using a phenomenological model for the turbulent dissipation based on wave reflection. We show that most of the dissipation for a given wave-frequency spectrum occurs in the lower corona, and the overall rms amplitude of the fluctuations evolves in a way consistent with observations. The frequency spectrum, for a Kolmogorov-like slope, is not found to change dramatically from the photosphere to the solar wind, however it does preserve signatures of transmission throughout the lower atmospheric layers, namely oscillations in the spectrum at high frequencies reminiscent of the resonances found in the linear case. These may disappear once more realistic couplings for the non-linear terms are introduced, or if time-dependent variability of the lower atmospheric layer is introduced.

3.2.1 Model Atmosphere

The chromosphere and the photosphere are modeled as a static layer, 2400 km thick, with the magnetic field organized in flux tube in supra-spherical geometry with constant temperature. The density varies almost exponentially and the magnetic field varies according to the flux tube expansion (A see below) in order to reproduce the properties of a coronal hole in the quiet Sun (Hollweg

Figure 3.11: From top left, clockwise: wind speed (solid line) and Alfvén speed (dotted line), numerical density, temperature and expansion factor as a function of heliocentric distance for the modeled atmosphere.
et al. 1982). Across the transition region the density falls off by two orders of magnitude, the wind passes from a speed of 0 km s\(^{-1}\) to 8 km s\(^{-1}\) while the magnetic field strength is continuous (about 10 G). The corona also expands supra-spherically and its temperature profile is chosen to fit observations (see fig. 3.11 and fig. 3.12 for a schematic representation): it starts at \(8 \times 10^5\) K at the coronal base, peaks at about \(3 \times 10^6\) K at 3 \(R_\odot\) and then falls off with distance as \(r^{-0.7}\) (Casalbioni et al. 1999). The wind speed profile follows from the continuity and momentum equations with given temperature and flux tube expansion, of the form \(A(r) = f(r)r^2\), with

\[
f = \frac{f_{\text{max}} \exp \left[ \frac{r_0 - r}{\sigma} \right] + f_1}{\exp \left[ \frac{r_0 - r}{\sigma} \right] + 1} \quad \text{and} \quad f_{\text{max}} = 1 - (f_{\text{max}} - 1) \exp \left[ \frac{r_0 - r_1}{\sigma} \right], \quad (3.12)
\]
3.2.2 Nonlinear Model

We choose the following model for the nonlinear terms

\[ \text{NL}_j^\pm = z^\pm(\omega_j) \frac{|Z^\pm|}{2 \lambda(r)} \]

(3.13)

where \( |Z^\pm| \) stands for the total amplitude of the Elsässer field integrated over the whole spectrum (\( \Omega \)) at the point \( r \), hence \( |Z^\pm| = \sqrt{\int_\Omega |Z^\pm(\omega)|^2 / \omega \, d\omega} \) (see fig. 3.13. This choice overestimates the transfer rate between high-frequency modes, for which the Alfvén effect is important (Shebalin et al. 1983). In reality the predominant interaction, as will be seen below, concerns the lowest frequency reflected mode and the full outward propagating spectrum, for which the resonance effects are not important.

The energy distribution among the modes influences the dissipation rate of all the waves coupled. In particular, at a fixed total rms energy, dissipation is reduced if the energy of the higher frequency waves is comparable to the lower
3.2. Application to the Sun Atmosphere and Solar Wind

frequency ones (flatter spectra) with respect to the case in which most of the energy is contained in the low frequency modes (steeper spectra) (Verdini et al. 2005).

Adding nonlinearities to the normalized equations for the wave propagation eqs. 2.37-2.38 one obtains (N and O indicate respectively normalized and original non-normalized variables)

\[
\frac{dz^\pm_N}{dr} - i \frac{\omega}{U \pm V_a} z^\pm_N - \frac{1}{2} \frac{V'_a}{V_a} z_N^\mp = - \frac{|Z^\mp_o|}{(U \pm V_a)2A} z^\pm_N
\]

(3.14)

\[
\frac{dz^\pm_N}{dr} + i \frac{\omega}{V_a} z^\pm_N + \frac{1}{2} \frac{V'_a}{V_a} z_N^\mp = \pm \frac{|Z^\mp_o|}{V_a2A} z^\pm_N
\]

(3.15)

for the corona and for the photosphere and the chromosphere respectively. Recall that the second, third coefficient in eqs. 3.14-3.15 represent the propagation (P) and reflection (R) coefficients already found in the linear analysis (respectively inverse of parallel wavelength, reflection scale height) while the last coefficient introduces the nonlinear dissipation (NL) coefficient (nonlinear length scale).

3.2.3 Boundary Conditions

Boundary conditions are chosen to assure an amplitude of the rms velocity field fluctuations (i.e. summed over the whole spectrum) of \( \approx 40 \text{ km s}^{-1} \) at \( 1 R_\odot \), as constrained by observations (Banerjee et al. 1998), with an assigned spectral distribution: this requires some trial and error since nonlinearity does not allow rescaling of the photospheric amplitude by simply rescaling values at the critical point \( X_a \). The shape of photospheric spectrum is imposed approximately thanks to the quasi-linear properties of the waves in the photosphere-chromosphere layer (small wave amplitudes) and the fact that transmission and nonlinearity yield frequency-independent evolution in the low corona, as shown in the next section. Given a slope \( p \) at the Alfvénic critical point, the transmission coefficient of the static layer \( T(\omega) \) (see eq. 2.30), can therefore be used to correct the initial spectrum \( |z'(\omega)| = |z'(\omega_0)| \times (\omega/\omega_0)^p \) to the desired spectrum at the photosphere imposing \( |z'(\omega)| = |z'(\omega_0)| \sqrt{T(\omega)} \times (\omega/\omega_0)^p \).

In order to describe the spectrum 32 modes are chosen in the range of frequency between \( 10^{-6} \text{ Hz} \) and \( 10^{-2} \text{ Hz} \) with increasing resolution at higher frequencies.

The phenomenological turbulent length scale varies as \( \lambda(r) = \lambda_0 \times \sqrt{A(r)} \), where \( 2\lambda_0 = 34,000 \text{ km} \) is imposed at the coronal base and corresponds to the average size of the supergranule. The waves are propagated from the Alfvénic critical point forward (to the Earth orbit) and backward (till the base of the
corona) by integration of eqs. 3.14. The conservation of the energy flux across the transition region allows one to determine the Elsässer fields below the discontinuity which are propagated back to the base of the photosphere using eqs. 3.15.

### 3.2.4 Results

Following Velli 1993 we compare the characteristic length scales of eqs. 3.14-3.15 in the two layers. First consider the thick lines in fig. 3.14 which represent the reflection and nonlinear coefficients (solid and dashed line respectively) normalized to the propagation coefficient for $\omega = 10^{-6}$ Hz, $10^{-4}$ Hz, $10^{-2}$ Hz (black, blue and green lines respectively) for a flat photospheric spectrum. Reflection has a maximum at the transition region and it falls off by a factor of about 100 in the corona (because of the density drop). The zeros in the reflection coefficient appearing for both the $z^+$ and the $z^-$ depend on the fact that $V'_a = 0$ (approximately in the corona), while the one located at $X_a$ appears only for the backward propagating waves since the propagation coefficient becomes infinite there (see eq. 3.14).

For the outward propagating wave (left panel) reflection is generally much greater (a factor 100) than dissipation in the photosphere-chromosphere and in the very low corona (below $\approx 1.2R_\odot$). Further out the nonlinear dissipation is smaller than reflection but of the same order of magnitude. For the inward propagating wave (right panel), again reflection dominates in the photosphere-
Figure 3.15: Normalized wave action density for the corona as a function of distance for 5 frequencies (10^{-6} Hz solid line, 10^{-5} Hz dotted line, 10^{-4} Hz dashed line, 10^{-3} Hz dotted-dashed line, 10^{-2} Hz triple-dotted-dashed line). The wave energy flux for the photosphere-chromosphere is plotted in the subpanel with the same line coding.

...chromosphere (by a factor of 10), but in the corona the dissipative coefficient is comparable or much greater than the reflection coefficient.

The relative dissipation of the linearly conserved quantities, as defined below in eq. 3.16, has hence different features in the two layers. In fig. 3.15 we plot the total wave action density for the corona (main panel) and the total wave energy flux for the static layer (sub panel) normalized to their base value for all the frequencies which form the spectrum, i.e.

\[
\frac{S^*(r, \omega)}{S^*_0(\omega)} = \frac{|z_N^+|^2 - |z_N^-|^2}{|z_{N_0}^+|^2 - |z_{N_0}^-|^2} =
\]

\[
1 - \frac{1}{2} \frac{dv}{Lv_a} \int_{r_0}^r \left( \frac{|Z_0^+|}{1 + M_a} |z_N^+|^2 + \frac{|Z_0^-|}{1 - M_a} |z_N^-|^2 \right),
\]

(3.16)

with the normalization eq. 2.36: the coefficients appearing in the integral are the nonlinear frequency integrated coefficients discussed above.

In the upper chromosphere the flux tube expansion is very rapid and reflection is strong, both the ingoing and outgoing wave contribute to the damping of the energy flux (comparable -less than one order of magnitude difference- nonlinear coefficient and wave amplitudes) and the relative dissipation is very high. Low frequency modes (lower plot in the subfigure) are the most damped (the most reflected) while high frequency modes (higher plots) are the less damped. In fact, inspection of eq. 3.16 reveals that the relative dissipation is quadratic in the frequency dependent wave amplitudes (|z_N|^2) which in turn increase with decreasing frequency because of the different reflection rate. In the corona, instead, beyond 2R⊙ the dissipation coefficient for the outgoing waves is weaker...
and their amplitudes grow, reflection is weaker as well, and an imbalance between outgoing and ingoing fluxes holds. Only the former contribute to the wave action density dissipation since now the dominant quadratic dependence in eq. 3.16 comes from the outgoing mode (see for example the approximate conservation form used by Cranmer & van Ballegooijen 2005). Note that for all frequencies the wave action density decreases at approximately the same rate. It turns out that the amplitude evolution is driven mainly by the nonlinear, frequency independent, term in the corona and by the reflection, frequency dependent, term in the photosphere-chromosphere, a feature we will find again studying the power spectrum evolution.

In comparison the heating rate per unit mass, an absolute measure of energy dissipation, integrated over the spectrum,

$$\frac{Q}{\rho} = \frac{Q^+}{\rho} + \frac{Q^-}{\rho} = \frac{|Z|^2 Z^+| + |Z|^2 Z^-|}{2d(r)},$$

(3.17)

is generally higher in the corona than in the photosphere-chromosphere, as shown in fig. 3.16. In the latter layer both the ingoing and outgoing wave contributes to the total amount of heating rate, while in the former most of the dissipation comes from the outgoing mode. The absolute dissipation is quadratic in the frequency integrated wave amplitudes and in the corona outgoing wave are allowed to grow almost undamped (low relative dissipation) but the existence of a small seed of ingoing wave assures a large absolute dissipation. This is not true in the photosphere-chromosphere, before the rapid expansion of the flux tube, where the wave amplitude is small and there is a small imbalance between outgoing and ingoing propagating wave amplitudes.

The effect of a different slope of the initial spectrum can be understood ana-
analyzing the contribution of each frequency to the nonlinear coefficient, plotted in thin lines in fig. 3.14. Starting with a flat photospheric frequency spectrum results in an approximately equal contribution to the total nonlinear term in the whole atmosphere, except for the outer corona where the nonlinear coefficient for the outward propagating wave is made up of essentially backward propagating waves at low frequencies. Note also that the frequency decomposed nonlinear coefficient is approximately the same for outgoing and ingoing propagating waves in the photosphere-chromosphere since reflection is high enough compared to dissipation. It follows that if a Kolmogorov-like photospheric spectrum \( P \propto \omega^{-5/3} \) is imposed, the nonlinear term is mainly made up of low frequency waves for both counter-propagating waves, in both the layers. This is can be seen in fig. 3.17 comparing the dashed thick lines and the solid thin lines: for \( \omega \gtrsim 10^{-4} \) Hz the contribution to the nonlinear coefficient is generally less then 10%. An exception is found below \( 2R_\odot \) for the outgoing mode, since reflection is high even for intermediate frequency wave (see fig. 3.19 for the photospheric layer). Note that a dip in the (frequency integrated) nonlinear coefficient for the outgoing mode appears below the location of vanishing reflection in both the photosphere and low corona, since the energy resides mainly in the low frequency mode.

This separate behavior in the two layers has strong consequences on spectral evolution. In fig. 3.18 the (compensated) total power in the fluctuations is plotted for different heliocentric distances. An almost Kolmogorov-like spectrum is imposed at the base of the photosphere with a procedure described at the end of section 3.2.3. At very low frequencies the spectrum practically does not evolve, in the whole domain, while there is a tendency to steepen at low-intermediate frequencies \( 10^{-5} \) Hz \( \lesssim \omega \lesssim 10^{-3} \) Hz. The behavior at
high frequencies is quite complicated. Some irregularities appear very close to the base of the photosphere and the overall tendency is that of flattening. Note, however, that most of the changes in the shape occur in the photosphere-chromosphere where the waves display a strong frequency dependent behavior. This makes the spectral evolution very similar to the linear case, below the transition region (except the energy level of the spectrum), and the appearance of the irregularities can be interpreted by means of the linear analysis. Accordingly, in fig. 3.19 we plot the transmission coefficient, defined in eq. 2.30, as a function of frequency for the photosphere-chromosphere. Note that the transmission is constant at low frequency, decreases at intermediate frequen-
cies and increase again at high frequencies, where several transmission peaks appear: basically all spectral evolution is qualitatively reproduced. The peaks originate from the discontinuity in the reflection scale height at the transition region (Velli 1993). In fact the amplitude of the reflected waves shows some nodes inside the domain and when their location coincides with the base of the photosphere the transmission is enhanced (a condition which depends on the frequency of the waves, see Hollweg 1978). When nonlinearities are introduced the location of the nodes depends also on the wave amplitude imposed at $X_a$ (see Verdini et al. 2005) and similarly if these nodes are located near the base of the photosphere the irregularities in the spectrum appear.

The slope of the spectrum imposed at the photosphere has negligible effects on the total power spectral evolution, however it changes the amount of energy residing in the ingoing and outgoing mode (or in the kinetic or magnetic fluctuations) at large distances and some constraints on the slope can be obtained using the available observational data. In fig. 3.20 the Elsässer energies $E^\pm$ integrated over the frequency spectrum are plotted (solid and dashed line respectively) along with the Ulysses and Helios data (Bavassano et al. 2000b), for a Kolmogorov (thick lines) and a flat (thin lines) initial slope with $\delta u = 40 \text{ km s}^{-1}$ at the coronal base. Both the data and the expected slopes are reproduced by the Kolmogorov-like photospheric spectrum while the flat one has too high outgoing energy and too low ingoing energy. The effect of high energy at high frequency waves is that of dissipating the inward wave, since they are mainly outward propagating: as a result outgoing waves are allowed to propagate almost undamped and their energy content is hence higher. Note that in the Kolmogorov case a dip, very close to the coronal base, appears as a signature of vanishing ingoing waves, a feature of the low frequency reflected waves. This results in a vanishing absolute dissipation (heating) which is not found for the flat case and has important consequences for the acceleration and heating of the solar wind.

In the following we consider only a Kolmogorov spectrum. In fig. 3.21 the root mean square amplitude of velocity field fluctuation integrated over the whole spectrum is plotted as a function of heliocentric distance (solid line, in dotted line we plot also the magnetic field fluctuation in velocity unit) along with some observational data (taken from Cranmer & van Ballegooijen 2005, to which we address for comment on the data set):

- Filled diamonds are non-thermal line broadening velocities measured by SUMER on the disk (Wilhelm et al. 1995),
- Crosses are non-thermal velocities derived from SUMER observations above the solar limb (Banerjee et al. 1998)
- The box represents the upper and lower limit given by Esser et al. 1999
Figure 3.20: Frequency integrated Elsässer energies as a function of heliocentric distance for a photospheric Kolmogorov spectrum with $\delta u = 40$ km s$^{-1}$ at the coronal base. Symbols indicate observational constraints (see text for explanation).

From UVCS off-limb data

- Stars are early measurements from Armstrong & Woo 1981
- The bars are recent measurements of transverse velocity field fluctuation using radio scintillation (Canals et al. 2002)
- Filled bars are the Helios and Ulysses data for the Elsässer energies, from Bavassano et al. 2000b, rewritten in term of the velocity field fluctuation assuming equipartition between magnetic and kinetic energy.

Note that the Helios and Ulysses data are obtained averaging over periods $\lesssim 1$ hour (corresponding to $\omega \lesssim 10^{-4}$ Hz) while all the other points in the figure refers to rms values. The overall agreement is quite good, even if data suggest a smaller power (more dissipation) just above the T.R. and more power (less dissipation) at about $2R_\odot$. Note that because of the equipartition assumption the Helios and Ulysses data disagree with the integrated quantities (the correct comparison has already been made above in fig. 3.20). As noted by Cranmer & van Ballegooijen 2005 the longitudinal velocity fluctuation data (filled diamonds) agree very well with the magnetic field fluctuation amplitudes (dashed line) and could indicate wave coupling among transverse and longitudinal mode. At leading order compressive effects are driven by the magnetic pressure originating from the incompressible fluctuations and represent a way for Alfvén waves to get rid of the energy excess above the T.R. If these compressional waves are isotropic and suffer some damping via shock formation, or other processes active in the low corona, they can reproduced the measured parallel $\delta u$ and supply the heating needed by current model of wind acceleration.
3.2. Application to the Sun Atmosphere and Solar Wind

Figure 3.21: Root mean square amplitude \( \delta u \) and \( \delta b \) (in velocity units) as a function of heliocentric distance for a photospheric Kolmogorov spectrum with \( \delta u = 40 \text{ km s}^{-1} \) at the coronal base. Symbols indicates observational constraints (see text for explanation).

3.2.5 Conclusions

In this work we have modeled the nonlinear evolution of Alfvén waves propagating through the photosphere, the corona and the solar wind till 1 AU. Nonlinear interactions occur between outward propagating and reflected waves, and it is assumed that a nonlinear cascade develops preferentially in a direction perpendicular to that of propagation, which we take to coincide with the direction of the mean radial magnetic field.

While the phenomenological nonlinear term acts as a dissipative sink for both outward and inward waves, independently of the wave frequency, reflection, provided by the stratification of the layer, is generally strong at low frequencies and decreases with increasing frequency.

We find that most of the heating occurs in the low corona (below the Alfvénic critical point), while very little power is dissipated below the transition region. For reasonable velocity field fluctuations at the base of the photosphere a sufficient amount of energy flux is transmitted through the transition region. The adopted frequency coupling is not able to reproduce the observed spectral slope and evolution in the Alfvénic range even though frequency integrated data at large distances constrain the outer spectrum to be steep (-5/3 slope). The modification of the frequency spectrum occurs mainly in the chromosphere and in the photosphere since waves experience a strong reflection at all the frequencies considered, while in the corona and the solar wind the spectrum maintains approximately the same shape one finds at the coronal base.
Nonlinear dissipation based on reflection acts in different ways depending both on the (ingoing and outgoing) wave amplitude and on the layer considered. In the corona, reflection is not very high but the outgoing wave amplitude is allowed to grow, so that the wave evolution is driven by the nonlinear interactions (all the modes evolve in the same way) and one finds a strong heating rate in the Sub-Alfvenic corona.

In the photosphere-chromosphere a strong reflection rate, combined with small wave amplitudes, leads to an evolution similar to the linear case, which depends on frequency, and a small heating rate.

As a result most of the wave energy dissipation takes place in the first 4 solar radii above the coronal base. The driving modes for dissipation are the modes which experience the biggest reflection, generally low frequency modes. However depending on the model of atmosphere, i.e. on its characteristic scale height, and on the energy distribution, i.e. flat or steep spectra, intermediate frequency modes can be important as well.

The spectral shape varies mainly below the transition region, it steepens at low-intermediate frequencies ($10^{-5}$ Hz $\lesssim \omega \lesssim 10^{-3}$ Hz) and flattens at high frequencies ($\omega \gtrsim 10^{-3}$ Hz) showing the characteristic features (energy peaks and frequency distribution) one finds in the transmission coefficient (linear behavior). In the corona instead, it maintains approximately the shape one finds beyond the T.R., because of the form of the nonlinear term adopted. The very low frequency range ($\omega \lesssim 10^{-5}$ Hz) practically does not evolve in the whole layer and it keeps the original slope at the photosphere. With this model of nonlinearities one can conclude that the spectrum one finds at 1 AU is basically the same spectrum at the base of the corona.

The input spectrum at the photosphere, whatever the shape is, is instead strongly modified by the transmission properties of the atmosphere below the transition region (independently of the model used for the nonlinear interaction). The energy peaks in the spectrum, resulting from an enhanced transmission at high frequencies, indicate that, even in presence of nonlinear interactions, the photospheric layer act as a filter for the energy injected through photospheric footpoint motion, if a smoothing of the forcing frequency is to be present, it must occur in this highly stratified layer.

The data at large distances suggest that energy at high frequency should be very low, however we find an energy increase at high frequency. Since the spectral evolution in corona depends also on the approximate frequency coupling contained in the nonlinear term, constraints on the photospheric input spectrum can not be given safely. Given that high frequency waves are transmitted through the T.R. and are quite energetic in the very low corona some other mechanism must be invoked to dissipate high frequency waves however a better modeling of the nonlinearities (as in sec. 4) will help in clarifying this issue.

As first pointed out by Hollweg (1981), such high frequency energy reservoir
can be the source for plasma heating processes operating in the low corona. Note that not only the peaks contribute to the energy budget, but the general flattening of the spectrum is important as well.

A comparison with measurements of $\delta u$ suggests that the model can be considered a very good approximation in the outer corona and solar wind while, despite the good agreement found in the low corona, some other processes must be invoked to reproduce the observed features below the Alfvénic critical point, such as compressible effects and wave coupling, especially in the chromosphere and photosphere (this issue is partially addressed in sec. 3.3).

Other models of turbulent transport have been constructed to fit the decay of turbulence with distance from the Sun in the solar wind beyond 1 AU (Smith et al. 2001; Breech et al. 2005) as well as to explain the extended heating in this region. Here the Alfvén speed can be neglected in the transport of the fluctuations, so that in some sense our model equations should be consistent with theirs, when rewritten in terms of the second order moments. A generalization of turbulence transport equations, consistent both in the corona, acceleration region and solar wind is a topic discussed in sec. 3.3 as well.
3.3 The Path Toward a Self Consistent Model of Coronal Heating and Wind Acceleration

The wave eqs. 2.26 are obtained under the assumption of stationary mean density, flow speed and temperature and then solved with these prescribed one dimensional equilibrium fields. Therefore they can not be used to follow the time dependent evolution of turbulent driven wind and only stationary solution can be found. Beside this limitation the prescribed mean fields are derived neglecting the effect of the time averaged wave pressure and reynolds stress in the momentum equation and, furthermore the turbulent heating in the energy equation, for which a temperature profile is actually assigned. An iterative procedure is now used to include the the back-reaction of the waves (turbulence) on the wind flow in order to find a stationary equilibrium solution for the turbulent heated wind, which, may exist although the iterative method adopted does not converge.

Actually, up to now, the incompressible turbulent heating alone (computed as in the previously introduced approximations) has not allowed any convergence to a solution for the coupled system. Convergence is easily reached when even a very small heating is supplied close to the Sun surface (at about $1.5 R_\odot$). We anticipate here that compressible effects, which has been neglected so far, could represent a viable source of the extra heating when subsonic compressible fluctuations are considered. However, we are still searching a procedure for their self-consistent inclusion in the turbulence equations to test the effect of compressible heating on the coupled system.

3.3.1 Wind and Wave Equations

The wind equations are derived in appendix A in the most general form allowed by the method of timescale separation. However steadiness of the equilibrium fields is assumed so that eqs. A.9-A.11 and eq. A.13, in which the wave contributions are retained explicitly, are used neglecting their time dependence. We neglect the evolution of the mean radial magnetic field which is actually assigned by prescribing the flux tube expansion $A(r)$. The large scale density, velocity and pressure equations are:

\begin{align}
\frac{1}{A} \frac{\partial (\rho U A)}{\partial r} &= 0, \quad (3.18) \\
\rho \left( U \frac{\partial U}{\partial r} \right) &= \frac{\partial p}{\partial r} - \rho \frac{GM_\odot}{r^2} + \left( \rho \frac{1}{2A} \frac{\partial A}{\partial r} \left( \frac{u^2}{2} - \frac{b^2}{4\pi\rho} \right) \right) - \frac{\partial}{\partial r} \left( \frac{b^2}{8\pi\rho} \right), \quad (3.19) \\
U \frac{\partial p}{\partial r} &= -\gamma p \frac{1}{A} \frac{\partial (UA)}{\partial r} + (\gamma - 1) Q, \quad (3.20) \\
\end{align}

\begin{align}
U \frac{\partial p}{\partial r} &= -\gamma p \frac{1}{A} \frac{\partial (UA)}{\partial r} + (\gamma - 1) Q, \quad (3.21)
\end{align}
3.3. The Path Toward a Self Consistent Model of Coronal Heating and Wind Acceleration

with

\[ Q = \rho \frac{1}{2} |Z^+|^2 |Z^-| + |Z^-|^2 |Z^+| \]  
\[ (3.22) \]

and \( \gamma = c_p/c_v \) as usual. The two last terms in the momentum equation 3.19 are the radial component of the Reynolds stress tensor (cfr. app. A), respectively the magnetic wave pressure and the ponderomotive forces, the latter vanishing in the WKB case. The heating function \( Q \) descends directly from the form of the nonlinear phenomenological model used in the wave eqs. 3.4 rewritten here in a compact form:

\[ (U \pm V_a) \frac{dz^\pm}{dr} - i \omega z^\pm + R^\pm_1 z^\mp + R^\pm_2 (z^- \mp z^+) = -\frac{z^\pm |Z^\mp|}{2 \lambda(r)}. \]  
\[ (3.23) \]

Here \( R^\pm_1 \) and \( R^\pm_2 \) are the reflection coefficients (along the fluctuation polarization and isotropic respectively) which depends on the large scale fields \( U, V_a \) and the expansion area \( A \). Recall that the wave equations above are valid (derived) only under the assumption of stationary mean density. However, in the following we will use a time dependent simulation to achieve a stationary equilibrium for the mean flow variables. More specifically the above wind equations are rewritten in conservation form, maintaining the time derivative, as:

\[ \frac{\partial (\rho A)}{\partial t} + \frac{\partial (\rho U A)}{\partial r} = 0, \]  
\[ (3.24) \]

\[ \frac{\partial (\rho U A)}{\partial t} + \frac{\partial (\rho U^2 A + p A)}{\partial r} = \frac{\partial A}{\partial r} p + \rho A a_w, \]  
\[ (3.25) \]

\[ \frac{\partial (\rho e_T A)}{\partial t} + \frac{\partial (U (\rho e_T A + p A))}{\partial r} = (\rho U a_w + Q) A, \]  
\[ (3.26) \]

for which we have grouped the radial acceleration due to “external” forces

\[ a_w = \frac{1}{2} A \frac{DA}{dr} \left( \frac{u^2}{2} - \frac{b^2}{4\pi \rho} \right) - \frac{1}{8\pi \rho} \frac{db^2}{dr} - \frac{GM_\odot}{r^2}, \]  
\[ (3.27) \]

and the internal energy is defined as

\[ e_T = e + \frac{1}{2} U^2 = \frac{1}{\gamma - 1} \frac{p}{\rho} + \frac{1}{2} U^2. \]  
\[ (3.28) \]

Note that the last term on the RHS of eq. 3.25 and the RHS of eq. 3.26 are forcing terms while, the first term on the RHS of eq. 3.25 is due to the expansion (i.e. non cartesian geometry). In deriving these equations a further assumption has been made, namely that the time averaged radial ponderomotive forces and pressure are computed using the small scale fluctuations \( u^2, b^2 \) derived from the rms amplitude of the wave solutions.

A third order Runge-Kutta method is used to advance in time the equations
while the spatial derivative are computed with an upwind first order scheme with a wave solver (see Laney 1998). At the top boundary, chosen to be at 1 AU, we require a free streaming supersonic flow and the density, speed and pressure are left free to change. At the bottom boundary (1 R⊙) instead we require a stationary temperature (pressure) and density which fix two of the characteristics of the euler equation, namely the characteristics propagating outward with speed $U$ and $U + C_s$. The third, free, characteristic propagates downward at speed $U - C_s$, its time evolution is hence computed using quantities inside the simulation domain and determines the wind speed at the Sun surface.

The procedure for the integration follows a scheme similar to that one used by MacGregor & Charbonneau (1994) who solved the linearized wave equations coupled to an equilibrium background field neglecting the energy equation. We start with an imposed temperature profile and $\gamma = \gamma_0$ and integrate the wind equations 3.18-3.19 in which the wave-derived quantities (ponderomotive force, magnetic pressure and dissipation) are set to zero. The solution obtained specifies the coefficient entering the “wave” equations which are in turn integrated. Boundary conditions are naturally imposed at the Alfvénic critical point ($X_a$) even if we require a given amplitude (say $\delta u_0 = 30$ km s$^{-1}$) for the velocity field fluctuation at 1 R⊙. Equations (3.23) are integrated backward in order to find the corresponding wave amplitude to impose at $X_a$, which is then used for the forward integration until 1 AU. After the first iteration step eqs. 3.24-3.25-3.26 are integrated simultaneously with $\gamma_{\text{iter}} = \gamma_{\text{iter}-1} + \epsilon_\gamma$, and the contribution of the wave-derived quantities are included. The resulting wind and Alfvén speed profile are then used again in the waves equations, and so on. When $\gamma = 5/3$ is achieved, the integration continues until convergence is reached, that is, when the relative variation of the quantities $U$, $V_a$, $T$, $Q/\rho$, $X_a$, $X_s$ is less than 0.01% (here, $X_s$ is the sonic critical point).

### 3.3.2 Heating Function

Current numerical simulations of wave driven solar wind include several mechanisms which are not present in our turbulent heated wind model, as described in section 2.2. In particular both Habbal et al. (1995) and Li et al. (1999) use separate equations to describe electrons and ions, focusing their attention on the anisotropic properties of the ion and electron temperature relative to the parallel and perpendicular direction, including self consistently ion-cyclotron resonances to achieve the dissipation of Alfvén waves. Besides the simplification of describing a single fluid flow, our model includes a self consistent way to account for the turbulent wave heating, usually described with ad hoc heating functions as in the previous works.

The incompressible heating function is plotted in fig. 3.22 (black line). Gen-
3.3. The Path Toward a Self Consistent Model of Coronal Heating and Wind Acceleration

Figure 3.22: Incompressible and gaussian heating function profile (in code units) in thin and thick line respectively, from 1 to 215 $R_\odot$, derived using the following parameters: $\delta u_0 = 30 \text{ km s}^{-1}$, $\omega = 0$, $Q_0 = 1$, $x_g = 1.7 R_\odot$, $\sigma_g = 0.5 R_\odot$ and $\lambda_0 = 0.02 R_\odot$. Open square on the incompressible heating function mark the sonic and Alfvénic critical points (from left to right respectively).

erally, its magnitude is determined by the amplitude of the fluctuations and the imbalance among the counter-propagating modes. Boundary conditions at $X_a$ impose a dominant outward flux, so that $E^+ > E^-$ in the whole domain. It turns out that the heating function is bounded from above by the level of $E^+$ when a not negligible seed of ingoing wave is present. On the other hand the vanishing (smallness) of one of the two species (the inward propagating waves in our case) stops the dissipation, setting the heating function to zero (small values). Note that this occurs close to the solar surface, the reason being a reduced (or zero) reflection rate.

It turns out that the incompressible turbulent heating alone, as modeled in eq. 3.22, do not allow to find a stationary solution for the mean flow, but additional heating must be supplied to reach convergence. We therefore introduce two forms of extra heating to be added as a compressible counterpart.

As a first approximation consider a gaussian function centered at $x_g$ and width $\sigma_g$ which directly yields the heating function close to the base, that is:

$$Q_{\text{gauss}} = \frac{Q_0}{\sqrt{2\pi}\sigma_g} \exp \left[ \frac{(r - x_g)^2}{2\sigma_g^2} \right]$$  \hspace{1cm} (3.29)

where $Q_0$ is used to control the amount of heating supplied.

As a further refinement we adopt a form derived in the context of nearly incompressible turbulence theory (Zank & Matthaeus 1991), assuming that the compressible heating is driven by the magnetic field fluctuations generated
by the Alfvén waves:

\[ Q_{\text{compr}} = \alpha_c \left( \frac{v_a^2 \vphantom{a}}{\sqrt{\vphantom{a} v_a^2 - c_s^2}} \right)^3 \frac{v_a^3}{A} \] (3.30)

with \( c_s \) and \( v_a \) being the sound speed and magnetic field fluctuation in velocity unit respectively, and \( \alpha_c < 1 \) is a parameter to control the entity of the compressible correction. For comparison the incompressible heating function 3.22 can be written as:

\[ Q_{\text{incomp}} = \alpha_0 f^\pm(\sigma_c) \frac{Z^3}{\lambda} \] (3.31)

where \( \sigma_c \) is the normalized cross helicity, \( Z = \sqrt{Z^{\pm2} + Z^{-2}} \),

\[ f^\pm = \frac{1}{2} \sqrt{1 - \sigma_c^2} \left[ \sqrt{1 + \sigma_c} \pm \sqrt{1 - \sigma_c} \right] \] and \( a_0 = 1 \). (3.32)

The additional heating is not computed in a self consistent way since it is not included in the wave equations as a dissipative term, therefore it should be kept low compared to the incompressible heating. Eq. 3.30 is singular for transonic fluctuations, a situation which manifest already at the first iteration step. We cannot use this expression as an extra heating added to the incompressible counterpart but it must be included in the wave-turbulence equations in order to test if divergence can be avoided².

In fig. 3.22 the incompressible and gaussian heating are plotted for a zero frequency \( z^\pm \) solution obtained in the starting step of the iteration (i.e. for nonlinear wave propagating in an atmosphere defined as in section 3.2 and shown in fig. 3.11, no back reaction of the wave being included to derive the equilibrium fields).

The gaussian heating is localized very close to the Sun surface, as prescribed, while the incompressible heating vanishes ad about \( 1.2 R_\odot \) and has a maximum very close to the sonic critical point (the first box in the plot). Despite its overall low level, the gaussian heating introduces a dominant extra heating close to the base.

### 3.3.3 Reference Data Set and Boundary Conditions

The data set used for comparisons with observations is partially the one used in section 3.2.4 (for the wave amplitude) with further observational constrains for the density, temperature and wind speed. This is the list of reference for the points shown in the results as observational constraints.

²We are working to this implementation in collaboration with W. H. Matthaeus
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- **Wind speed**: values inside 1 AU are deduced from radio scattering measurements of south polar streams (Grall et al. 1996). The wind speed at 1 AU is instead obtained from in situ measurements of Ulysses at 45° S latitude (McComas et al. 2000).

- **Density**: values inside 1 AU are obtained from white light coronal observations on SPARTAN 201 and from measurements of the non-thermal line broadening of forbidden lines of Si with SUMER on SOHO (Fisher & Guhathakurta 1995; Banerjee et al. 1998 respectively). The value at 1 AU is again obtained reducing Ulysses in situ measurements (McComas et al. 2000).

- **Normalized cross helicity**: the level of cross helicity beyond 1 AU is obtained from Ulysses in situ measurements of the Elsässer energies reduced and studied by Bavassano et al. (2000b,a) and given by Breech et al. (2005).

- **Temperature**: values inside 1 AU are upper and lower limits on the perpendicular temperature of neutral hydrogen derived from the line-width measurements of H I Lyα in coronal holes (Kohl et al. 1998). Values beyond 1 AU are again Ulysses in situ measurements (Breech et al. 2005).

- **Turbulence Level**: values beyond 1 AU are obtained combining velocity and magnetic field fluctuation measurements collected in the National Space Science Data Center (NSSDC) Omnitape data set (Smith et al. 2001).

Except when explicitly mentioned, all the runs have been done with the following “standard” parameters.

- **Wave parameters**: rms velocity fluctuations at the base, $\delta u_0 = 30 \text{ km s}^{-1}$; frequency, $\omega = 0 \text{ Hz}$, turbulent correlation length scale at the base, $\lambda_0 = 0.02 \text{ R}_\odot$.

- **Wind parameters**: density at the base, $n_0 = 5 \times 10^8 \text{ cm}^{-3}$; temperature at the base, $T_0 = 4 \times 10^5 \text{ K}$; magnetic field at 1 AU, $B_{1\text{AU}} = 3 \text{ Gauss}$; flux tube expansion parameters (see eq. 3.12), $f_{\text{max}} = 7.26$, $r_1 = 1.31 \text{ R}_\odot$, $r_0 = 1 \text{ R}_\odot$ and $\sigma = 1 \text{ R}_\odot$ (we consider an expansion occurring on a wider range than that given by Munro & Jackson 1977).

### 3.3.4 Pure Incompressible Heating

The pure incompressible case does not yield a stationary solution for the coupled wind - wave system of equations. The initial condition, i.e. the first
integration in the iteration, is shown in fig. 3.23 in orange lines and it is derived imposing a temperature profile which fits observational constraints. All the other plots (wind speed, density, cross helicity and turbulence level - or equivalently the total energy in the fluctuations $E^+ + E^-$) well agree with the data shown. In particular, beyond 1 AU, the data relative to the turbulence level and the normalized cross helicity are well reproduced by nonlinear turbulent waves propagating in a wind with constant speed of about $800 \text{ km s}^{-1}$ (these are the solutions found in the previous section 3.2).

However, as the system is evolved, oscillations in the location of $X_a$ and $X_t$ appears, instead of convergence, as shown in fig. 3.24. Note the appearance of solutions with multiple sonic critical points from iteration 9 and beyond. Before this multiple critical point behavior, the temporary solution already shows profiles of flow speed, density, temperature, turbulence level and normalized
cross helicity which are remarkably different from the initial conditions, as shown in fig. 3.23 for $n_{\text{iter}} = 8$ (black line). In the inner corona the normalized cross helicity is very close to unity, indicating that inward propagating waves are almost completely damped. The turbulent energy ($E^+ + E^-$) is much larger than the initial value, since the outward propagating waves are left free to grow in absence of an efficient turbulent dissipation.

The heating function producing the solution at $n_{\text{iter}} = 8$ is plotted in the bottom panel of fig. 3.25 (black line). In the inner corona ($R < 3R_\odot$), it is bounded by the low level of ingoing mode, the zeros corresponding to vanishing inward waves, and further out it increases following the grow of the outward mode $E^+$ sustained by a saturated level of inward wave amplitude. Accordingly the temperature profile has a local maximum close to the base followed by a local minimum (vanishing heating function). Most of the heating occurs between the sonic and Alfvénic critical points due to increase of the wave amplitudes. Note, however, that despite the very low (but not vanishing) value of the heating function, the temperature increases very rapidly to the local maximum (since density is higher at the base).

In the top panel of fig. 3.25 the acceleration exerted by the wave on the wind is also plotted (black solid line for $n_{\text{iter}} = 8$) to be compared to the initial condition (orange solid line). In contrast to to the initial wind, which is mainly accelerated indirectly by the heating, the wind found in this iteration is accelerated directly by the wave as well. The magnitude of direct acceleration is large in the whole domain, overcoming gravity as close as $3R_\odot$. The non WKB acceleration (the pondemorotive forces, blue line) adds a small correction to the magnetic pressure which sustains the wind in the whole domain. Eventu-
Figure 3.25: Top panel: Net acceleration exerted on the wind by the wave for $n_{iter} = 0, 8, 9$ (solid line, black, orange and blue respectively). The different contribution to the acceleration for $n_{iter} = 8$ are also shown: gravity ($g$), magnetic pressure ($\nabla P_m$) and the ponderomotive forces ($acc$). Bottom panel: Heating function for $n_{iter} = 0, 8, 9$, orange, black and blue line respectively.

ally it could play an important role in balancing the magnetic pressure itself, making possible to attain a constant wind speed beyond 10 $R_\odot$. The resulting wind has low speed in the inner corona (inefficient heating) but has a terminal speed twice the initial terminal speed. Finally the density drops of about an order of magnitude at 1 AU following the continuity equation.

The multiple critical point temporary solutions (for an example see the blue lines in fig. 3.23) arise naturally from the “evolution” of the system. Increasing position of the sonic critical point in the first iteration steps reflects the fact that the temperature drop caused by the low level of turbulent dissipation is moving outside from the base. However, the heating function is not smooth and displays local maxima and minima which introduce small scale gradients in the wind and Alfvén speed (the latter being determined by the density variations). This in turn increases reflection and hence the ripples in
3.3. The Path Toward a Self Consistent Model of Coronal Heating and Wind Acceleration

The heating function are amplified as the iteration proceeds (cfr. blue line in fig. 3.25). Note that this happens close to the base (generally below $4\,R_\odot$), before the large rise of the heating function which, instead, is pushed outside. At a certain point ripples are so high that the wind is heated from below, accelerating faster and reaching the first sonic point (the wind becomes supersonic) at smaller distance than the previous iteration. However, the ripples in the heating function decreases further out (cfr. the blue line between $3\,R_\odot$ and $4\,R_\odot$ in the bottom panel of fig. 3.25) and temperature decreases while the direct acceleration partially sustains the wind in this region: as a result the wind reaches the second sonic critical point (becoming subsonic) in the next, large, increasing branch of temperature (the part of the heating function driven by the wave amplitude increase), experiencing a sudden drop. Further out (above $\sim 4\,R_\odot$) the direct acceleration allows to pass the third sonic point (the wind becomes supersonic again) even though temperature is still increasing. The next temporary solution is strongly modified because of the nature of the second sonic point. In fact, the density (and hence the Alfvén speed) and the wind speed show a shock-like behavior, jumping suddenly across $X_s$, causing a further increase of the reflection rate, so that the waves experience a strong dissipation. The resulting wind is hence again heated from below, however, (generally) the ripples in the heating function have increasing amplitude and the temperature rise on average, avoiding multiple sonic points despite local drop of temperature occurs. Accordingly, in the iteration path, multiple sonic critical point solutions are followed by standard single sonic critical point solutions (cfr. fig. 3.24). Exceptions are found at $n_{\text{iter}} = 27$ and $n_{\text{iter}} = 49$ for which the jumps in the wind and Alfvén speed are not big enough to supply, through reflection, the heating required to maintain a rising temperature on average. Note also that these sequences of multiple sonic point solutions yield the lower sonic points in the iteration, a direct consequence of the occurrence of the two successive shock-like triggered heating events.

3.3.5 Gaussian Heating

The gaussian artificial heating is added for two main purposes: from one side it allows the convergence of the method, which has not been attained so far; on the other hand it helps in (partially) solving the mass flux problem, i.e. reasonable value for the density at 1 AU are obtained. The drawback is of course the lack of self consistency of the heating model. In fig. 3.26 two solutions obtained with different parameters for the gaussian function are plotted along with the initial condition (the orange line). The two solutions differ for the position of the center of the distribution and its width, namely

- a: $x_g = 1.7\,R_\odot$, $\sigma_g = 0.5\,R_\odot$ and $Q_0 = 1$ (black lines),
- b: $x_g = 1.3\,R_\odot$, $\sigma_g = 0.25\,R_\odot$ and $Q_0 = 1$ (blue lines).
The convergence of the iterations is attained thanks to the smoothing effect caused by the *ad hoc*, non-vanishing, heating function close to the base. It in fact inhibits the amplification of the ripples characteristic of the incompressible case and the resulting temperature, wind speed and density are regular solutions without sudden jumps or drops in their profiles. Note that, depending on the location of the gaussian center, the temperature profile may display or not a local minimum. If this is the case (b), the wind speed and the density well agree with observational constraints in the outer heliosphere since the incompressible heating sustains both the wind acceleration and the temperature increase through reflection, although the wind acceleration close to the base is still too low. When, instead, the gaussian is closer to the base (a), the overall temperature level is quite low, the wind accelerates faster in the inner corona but its terminal speed is only about 500 km s\(^{-1}\) and the density is larger than the observational constraints. Since the wind and temperature solutions are
3.3. The Path Toward a Self Consistent Model of Coronal Heating and Wind Acceleration

Figure 3.27: Mass flux in arbitrary unit as a function of distance for \( n_{\text{iter}} = 9 \) in the pure incompressible heating case (black line) and for the converged solutions with extended gaussian heating, case b (orange line).

Even smoother than in the case b, the reflection rate is partially suppressed and the resulting incompressible turbulent heating is too small to sustain the wind acceleration in the outer corona.

3.3.6 Discussion

Several simplifications have been made to construct the model, among them the phenomenological turbulent dissipation causes the most severe problem in the convergence to a unified solution of the coupled wind and wave system. In fact, it produce a heating function with local maxima and vanishing minima close to the base which introduces smaller length scale in the system, enhancing the reflection rate and the turbulent heating locally. When a sufficient amount of internal energy is supplied at the base the wind accelerates, temperature increases but their profiles are smoother in the inner corona, reflection is hence suppressed and a new cycle begins.

However, forcing the convergence with artificial, not self consistent, heating reveals the complexity of the reflection driven mechanism. In particular the reflection rate is a sensible parameter, that is its variation may produce big changes in the profiles of the (converged) solutions. Heating at the base suppresses reflection and the turbulent heating is not able to accelerate the wind to its terminal speed and to produce an acceptable temperature in the outer heliosphere. It suffices to push out a little bit the ad hoc heating function to obtain an amount of reflection capable of driving the wind and to increase the temperature in a way consistent with the observational constraints.
The desired “missing” heating function should scale with the inverse of temperature in the inner corona and vanish at large distance. At the moment a form of compressible heating driven by the magnetic pressure and ponderomotive forces is the best candidate. However, to clearly understand the role of compressible heating relative to the convergence and the profile of the solutions, the inclusion of this sink in the wave equations is mandatory (but not straightforward) in light of the “non result” obtained in the incompressible case.

Still remains an issue to be discussed, pertaining the time dependent euler equation solver. The multiple critical points solutions display a jump discontinuity when the wind approaches the sonic point from above, i.e. when it becomes subsonic through the jump. The code used to integrate the equations is built on a simple numerical scheme which, actually, is not able to capture shocks, such as the one supposed to be formed in above solutions. In fig. 3.27 the mass flux $\left(\rho U A\right)$ is plotted for $n_{\text{iter}} = 8$ (black line) and for the converged solution with gaussian heating. In the stationary solution we are looking for, this quantity should be time independent (by definition) and uniform, in virtue of the continuity equations (cfr. eqs. 3.24-3.25). It is quite evident that uniformity is not reached in the low corona (before $3 R_\odot$) and more important at the second sonic critical point. The grid used in the numerical scheme is not uniform and probably this is the cause for the non-uniformity in the low corona. At the critical point, the numerical scheme clearly fails in satisfying the Rankine-Hugoniot jump conditions, making questionable the results obtained for the pure incompressible case and probably also avoiding the convergence of the method, although the run away process described above seems not to originate from the appearence of jumps at the critical point.
Modelling Nonlinear Interactions

In this part of the thesis the back reaction of the waves on the wind is neglected in favor of a more accurate description of the nonlinear couplings in the waves equations. In fact, the phenomenological term used so far imposes the most severe limitations in studying the turbulent dissipation and completely neglects the dynamics in the perpendicular plane, where turbulence is assumed to develop. The dependence of the fluctuating fields on the perpendicular coordinate is modeled with an MHD version of a 2D shell model. Despite we lose information on the structures forming in the perpendicular plane (we use a scalar wavevector to describe the dynamics on a plane), this low dimensional model allows to span several decades in the perpendicular wavenumbers, the effective Reynolds number of the simulation is in fact very high making this kind of model suitable for studying turbulent phenomena.

Applications will be given for the solar chromosphere and for the solar corona (with wind) separately. For the former an isothermal atmosphere in hydrostatic equilibrium is assigned while for the latter a temperature profile is imposed to derive the wind in which the waves propagate. The model atmospheres are exactly the same described in sec. 3.2 (with the exclusion of the transition region that joined the two layers). The main novelty is that now turbulent spectra in the perpendicular wavenumber can be obtained, along with the frequency spectra. Moreover the heating function is the result of a cascade process which transfers the energy to small scales, at which the dissipative coefficient is no more negligible. In other words the nonlinear timescales are properly taken into account in the dynamical evolution of the system.

It is found that both in the chromosphere and in the corona the perpendicular spectra in the inertial range can be described by a power law function, as expected for turbulence, and their slope varies with distance. Moreover, the time averaged heating function in the low corona does not show the large scale ripples as in the phenomenological model discussed in sec. 3.3.4 and its level is
generally lower, with consequences on the existence of a stationary solution for the coupled wind and wave system. Notably the heating rate is very high in the chromosphere: despite the fluctuations have small amplitudes, the reflection rate is high and nonlinear interactions are efficient in developing a turbulent cascade.

4.1 Turbulence Model and Numerical Implementation

The three dimensional domain is decomposed along the radial direction in a series of planes perpendicular to the mean magnetic field. In each plane the nonlinear interactions are described by a simplified model, the shell model, able to capture the main features of the MHD turbulent dynamics. The shell model used in the planes is a 2D version of the MHD shell model (Glaouguen et al. 1985; Biskamp 1994; Giuliani & Carbone 1998; Frick & Sokoloff 1998), derived as a generalization of the originally introduced hydrodynamical “GOY” model (Gledze-Ohkitani-Yamada, Gledzer 1973; Yamada & Ohkitani 1987, 1988b,a), and more recently used in RMHD simulations of solar coronal loops (Nigro et al. 2004; Buchlin et al. 2004; Buchlin & Velli 2006).

The fluctuating fields in each plane are fourier decomposed in the perpendicular wavenumbers. The corresponding 2D fourier space is divided in concentric shells so that a given \( k_\perp \) belongs to the shell \( S_n \) delimited by the circles of radius \( k_n \) and \( k_{n+1} \), the radius scaling as \( k_n = k_0 \lambda^n \). In each shell, two complex scalars, \( u_n \) and \( b_n \) are assigned, representing the velocity and magnetic field increments \(|u(x + l) - u(x)|\) and \(|b(x + l) - b(x)|\) at the scale \( l = 2\pi/k_\perp \) which belong to the shell \( S_n \). For convenience the magnetic and velocity fluctuations are rewritten in term of the Elsässer fields \( Z_\pm^n = u_n \mp b_n / \sqrt{4\pi \rho} \) (for a magnetic field pointing outward), the shell decomposition being equivalent. The nonlinear interactions occur between wavenumbers of the same order of magnitude (local interaction approximation) and more specifically only between three consecutive shells, i.e. the triads \((n - 2, n - 1, n)\), \((n - 1, n, n + 1)\) and \((n, n + 1, n + 2)\) (see fig 4.1 for a schematic representation of the nonlinear interactions).

The fluctuation eqs. 2.26, implemented with the shell model for each plane on \( r \), becomes

\[
\frac{\partial Z_\pm^n}{\partial t} + (U \pm V_a)\frac{\partial Z_\pm^n}{\partial r} + \left[R_\text{iso}^\pm\right] Z_\pm^n + \left[R_\text{pol}^\pm \pm R_\text{iso}^\pm\right] Z_\mp^n = -k_n^2 \left(\nu^+ Z_\pm^n + \nu^- Z_\mp^n\right) + ik_n(T_{n}^\pm)^*, \tag{4.1}
\]

where the complex Elsässer fields, \( Z_\pm^n \), depend on time and on the radial coordinate \( r \). In the RHS \( \nu^+ = 1/2(\nu + \eta) \) are the dissipative coefficients, \( (T_{n}^\pm)^* \) is the nonlinear interaction term for the shell \( n \), while in the LHS the reflection terms are grouped in diagonal and off diagonal terms with the reflection along
the field polarization ($R_{pol}$) and isotropic reflection ($R_{iso}$) given by:

$$R_{pol}^\pm = \frac{1}{2} \int \frac{dA}{dr} (U \mp V_a), \quad (4.2)$$

$$R_{iso}^\pm = \frac{1}{2} \left[ \left( \int \frac{dA}{dr} + \frac{d}{dr} \right) \left( V_a \mp \frac{1}{2} U \right) \right]. \quad (4.3)$$

Since only counter-propagating waves interact nonlinearly the above couplings reduce to 6 nonlinear terms, giving the equations which constrain the parameter of the model. Their derivation is given in appendix B and is based on a few more assumptions, namely that nonlinear interactions conserve the three (ideal) 2D invariants (energy, cross helicity and ananostrophy, the latter corresponding to the magnetic helicity in 3D) when dissipative coefficients are neglected. The form of the nonlinear terms is finally:

$$T_n^\pm = \frac{\delta + \delta_m Z_{n+1}^\pm Z_{n+2}^\mp}{2 \lambda^2} + \frac{2 - \delta - \delta_m r_{n+1}^\pm Z_{n+2}^\pm}{2 \lambda^2}$$

$$+ \frac{\delta_m - \delta Z_{n+1}^\pm Z_{n-1}^\pm}{2 \lambda^2} + \frac{\delta + \delta_m Z_{n+1}^\pm Z_{n-1}^\pm}{2 \lambda^2}$$

$$+ \frac{2 - \delta - \delta_m r_{n-1}^\pm Z_{n-2}^\pm}{2 \lambda^2} \quad (4.4)$$

with $\delta = 1 + \lambda^{-\alpha}$ and $\delta_m = -\lambda^{-\alpha}/(1 - \lambda^{-\alpha})$ (For the 2D case, $\alpha = 2$). It is a usual choice to set $\lambda = 2$ for the shell decomposition of the plane, even if other values can be chosen (see Biferale 2003 for comment on this point and Giuliani
4. Modelling Nonlinear Interactions

The fluctuating field eqs. 4.1 are advanced in time using a third order Runge-Kutta method. The spatial derivative appearing as the second term in the LHS (advection term) is computed using a second order, explicit numerical scheme inside the domain, the Fromm method (Laney 1998). Generally this method is obtained as a weighted average of the Beam-Warming scheme (a second order explicit upwind scheme) and the Lax-Weindroff scheme (a second order explicit centered scheme): for every timestep four stencils (2 upwind and 1 downwind) are used to compute the spatial derivative (The two separated methods are used instead at points next to the boundaries). The scheme is useful in the current application since it conserves the wave phase, a quantity of particular interest when considering the interactions among reflected waves.

The numerical algorithm is written in Fortran 90, the explicit scheme for the wave propagation allows an efficient parallelization along this direction, each processor acting only on an interval of planes and communications are restricted to the points at the boundary of each subdomain. The code is largely discussed by Buchlin (2004), even though some new implementations have been done (namely open boundary conditions, stratification in the radial direction, inclusion of a mean flow).

The following definitions will be used in next section and are collected here for reference:

- energy density \( \varepsilon_w = \frac{1}{4} \rho \sum_n \left( |Z^+_n|^2 + |Z^-_n|^2 \right) \) (4.5)
- energy \( \varepsilon_w = \frac{1}{4} \sum_{n,r} \rho_r A_r \left( |Z^+_{n,r}|^2 + |Z^-_{n,r}|^2 \right) = \sum_r A_r \varepsilon_w \) (4.6)
- energy flux \( \phi_w = \frac{1}{4} \rho \left( V_a \sum_n \left( |Z^+_n|^2 - |Z^-_n|^2 \right) + U \sum_n \left( |Z^+_n|^2 + |Z^-_n|^2 \right) \right) \)
- net energy flux \( \phi = A(r) \phi_w \cdot \hat{r} \big|_{top} - A(r) \phi_w \cdot \hat{r} \big|_{bot} = \phi_{top} - \phi_{bot} \) (4.8)
- dissipation \( Q = \frac{1}{2} \rho \sum_n \left[ \nu k^2_n \left( |Z^+_n|^2 + |Z^-_n|^2 \right) \right] \) (4.9)
- total dissipation \( D = \sum_r A_r Q_r \) (4.10)
- normalized cross helicity \( \sigma_c = \sum_{n,r} \frac{|Z^+_{n,r}|^2 - |Z^-_{n,r}|^2}{|Z^+_{n,r}|^2 + |Z^-_{n,r}|^2} \) (4.11)

in which \( \eta = \nu \) is assumed, the index \( n \) refers to the shell number and the index \( r \) to the position along the radial direction. Eq. 4.1 is rewritten in nondimensional form adopting the same normalization used in chapter 3:
4.2 Turbulence in Open Magnetic Structures

In the following we describe the results obtained for two simulation setups. The first represents a chromospheric layer (with the exclusion of the transition region) which is modeled as an isothermal, static layer, subject to a radial gravitational field and with a cross section which differs from that given by radial geometry by a factor of about 100. The second setup represents the low corona (starting above the transition region) and solar wind (until $50 \, R_\odot$). A non-uniform (imposed) outflow is considered and the cross section expands radially in the overall domain, except in the inner corona where the expansion deviates from the radial one by a factor of about 7.

As discussed in sec. 2.3.3 for the definition of the transmission coefficient, correct boundary conditions are imposed if the waves are free to leave the domain without reflection at the top boundary. However, in view of a time dependent simulation, we deal with a different problem at the boundaries. For the moment assume that the system is forced at the bottom boundary, were it at the photospheric or coronal base, and consider instead the top boundary. Here one has to know, or impose, the incoming waves entering the domain, or strictly speaking the inward flux. For the coronal layer it suffices to extend the simulation domain in order to include the Alfvénic critical point. In fact, beyond this distance the waves are advected by the wind. For a reference frame positioned on the Sun (as in our case) all the waves propagate outward. To update the $z^-$ at the top boundary one needs only to compute the fields at the points belonging to the internal simulation domain, therefore an upwind scheme is particularly useful (note that at $X_a$ the stencils of an upwind scheme for $z^-$ reverse in the spatial direction, its phase speed changing sign). For the static layers, instead, the $z^-$ always propagates inward. Imposing an inward wave at the top the domain, no matter if feasible, will result in a partial reflection of the outgoing mode. Therefore, to assure a transparent boundary, two conditions are required: no incoming waves ($z^- = 0$) and a uniform Alfvén speed at the top boundary (the latter meaning a vanishing reflection rate, for consistency with the former condition).
Figure 4.2: **Bottom panel**: power spectrum of the total energy in the forcing, i.e. summed over the shells $n_f = 1, 2, 3$ in which the forcing is applied, for $\tau^* = 10000$ s (in arbitrary unit, values on the y axis are the logarithm of the power spectrum). **Top panel**: time series of the applied forcing (in arbitrary unit).

At the bottom boundary the system is forced imposing a fluctuating field, $\psi$, which varies with time. The forcing shells are generally chosen to be $n_f = 1, 2, 3$ among a set of 21 shells which describe the perpendicular wavenumber space ($n_{\text{shell}} = 0, 20$). The functional form of the forcing is:

$$\psi_{n_f}(t) = \psi_f \left[ e^{2i\pi A_{n_f}} \sin^2(\pi t/\tau^*) + e^{2i\pi B_{n_f}} \sin^2(\pi t/\tau^* + \pi/2) \right],$$  \hspace{1cm} (4.12)

where $\psi_f$ is the amplitude of the forcing, $\tau^*$ is a characteristic timescale, given as an input parameter, and $\mathcal{A}$ and $\mathcal{B}$ are random phases which change every time the corresponding $\sin^2$ term vanishes\(^1\) and are constant in the meanwhile. This function has a correlation time of the order of $\tau^*$ and its power spectrum is shown in fig. 4.2, along with the corresponding time series. For large value of $\tau^*$, long living, low frequency fluctuations are injected; conversely, for small correlation times, the forcing is made of several high frequencies wavepackets of short duration. As a result the spectrum is approximately horizontal for $\nu < (2\tau^*)^{-1}$, while for $\nu > (2\tau^*)^{-1}$ it approaches to a power-law scaling with slope equal to $-5/2$ (steep decrease). Note that a considerable amount of energy is injected at frequencies below the correlation time in the form of uncorrelated fluctuations.

\(^1\)At $m\tau^*$ for $\mathcal{A}$ and at $(m+1)/2\tau^*$ for $\mathcal{B}$, with $m$ integer.
4.2. Turbulence in Open Magnetic Structures

Table 4.1: Parameters for the specified atmosphere. The chromosphere is an isothermal static layer in hydrostatic equilibrium. The corona is permeated by a wind with solar like properties, the stratification is obtained solving the continuity and momentum equation with an imposed temperature profile which fits observational constraints. $L$ is the radial extent, $n_0$, $T_0$, $B_0$, $V_{a0}$, $U_0$ indicate respectively the numerical density, temperature, magnetic field, Alfvén and wind speed at the base. $\tau_{cr}$ is the crossing time and $\tau_{ref}^{\min}$ is the minimum reflection timescale.

### Table 4.1

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<th>$T_0$ (°K)</th>
<th>$B_0$ (G)</th>
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### 4.2.1 Time Scale Ordering

Several timescales determine the dynamics described by eq. 4.1, which, on its own, does not assure the development of a turbulent cascade. One can identify the following characteristic times:

- $\tau_{al} = \delta r/(V_a \pm U)$, the time taken by a wave to propagate to the next grid point in the domain, the Alfvén timescale,
- $\tau_{cr} = \int_{r_{\text{bot}}}^{r_{\text{top}}} \delta r/(V_a \pm U) \, dr$, the crossing timescale,
- $\tau_{ref} \sim |R_{\text{iso}}|^{-1} \sim |R_{\text{pol}}|^{-1}$, the reflection timescale,
- $\tau_f = \tau^{\ast}$, the time associated to the forcing frequency (or the correlation timescale), the forcing timescale,
- $\tau_{nl}(k_n) \sim |k_n z^\pm|^{-1}$ the nonlinear timescale at the $n$ shell,
- $\tau_0 \sim |k_n z^\pm|^{-1}$ the forcing nonlinear timescale,
- $\tau_d \sim |\nu^2 k_n^2 z^\pm|$ the dissipation timescale associated to $\nu$ or $\eta$,

which all vary with the position $r$ considered.

The first three timescales depend on the properties of the specified atmosphere (density, magnetic field, flux tube expansion and radial extent), for our model atmosphere\(^2\) (see fig. 3.11 in sec. 3.2.1), $\tau_{cr} \approx 500 \text{ s}$ and the minimum $\tau_{ref} \approx 4 \text{ s}$. Clearly the Alfvén timescale is controlled by the resolution adopted in the simulation, its main influence is on the numerical dissipation and stability of the code and it determines the smallest frequency that can be kept. This

\(^2\)We refer here to simulations for the chromospheric layer, analogous reasoning applies to the corona. See table 4.1-4.2 for a complete reference.
### Table 4.2: Parameters for the simulation.

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<th>(\tau^*) (10(^3) s)</th>
<th>(\psi) (km/s)</th>
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<th>(n_f)</th>
<th>(\lambda_0) (km)</th>
<th>(\tau^*) (10(^3) s)</th>
<th>(\psi) (km/s)</th>
<th>(\nu) ((R_\odot u^*))</th>
<th>(t_{\text{sim}}) (10(^3) s)</th>
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Table 4.2: Parameters for the simulation. **Legend:** L and R refer to linear and nonlinear runs in the chromospheric layer, W to nonlinear runs in the corona; \(n_r\), number of planes in the radial direction; \(n_{\text{shell}}\), number of shells in each plane; \(n_f\), forcing shells; \(\lambda_0\), biggest perpendicular length scale at the base; \(\tau^*\), forcing period for plane wave forcing or correlation timescale for the time-correlated forcing; \(\psi\), kind of forcing applied; \(t_{\text{sim}}\), total simulation time. The forcing is applied on the velocity fluctuation or on the outward propagating wave, indicated respectively with the legend \(u\) and \(z\) in the \(\psi\) field of the table, the value of the applied forcing is indicated in the same box; when a plane wave forcing is considered a pedix \(w\) is added to the corresponding field. In the chromosphere the amplitude of the forcing is tuned to have a rms value of velocity fluctuations at the top of the domain of about 33 km s\(^{-1}\) for the nonlinear runs. In the corona the amplitude of the forcing is set to obtain the same reference value at the base of the domain. 

is generally the smallest timescale in the whole domain, for \(n_r = 2000\) the minimum \(\tau_{al} \approx 8 \times 10^{-3}\) s and assures a low numerical dissipation compared to the turbulent dissipation with reasonable computational cost. Recall that reflection couples linearly the counter-propagating Elsässer fields, for small value of \(\omega \tau_{ref}\), i.e. small frequencies compared to the reflection timescale, the resulting reflection rate can be so high to force the normalize cross helicity to \(\approx 0\) (equal amount of outward and inward waves). 

The nonlinear, nonlinear forcing and dissipative timescales depend on the pa-
rameters of the simulation concerning the turbulence properties, namely, the forcing shells, the smallest perpendicular wavenumber, the number of shells and the dissipative coefficients (see tab. 4.2). The forcing timescale is set to $\tau^* = 1000$ s, so that most of the energy is injected only at low frequencies $\nu < 1/2\tau^*$. The smallest perpendicular wavenumber is constrained by the average dimension of the granular motions at the base of the photosphere (the bottom boundary of the simulation domain), $\lambda_0 \approx 34000$ km. Finally the dissipative coefficient is arbitrarily chosen in order to obtain a wide inertial range at all distances with a number of shells $n_{sh} = 21$. The extent of the inertial range depends on the dynamical evolution of the system, we restrict to the case $\nu = \text{const}$: since the $k_\perp$ range moves to smaller wavenumber with increasing $r$, the dissipative scale (given by the relation $\tau_d(k_n) = \tau_d(k_n)$) decreases and the inertial range expands with distance as will be shown in sec. 4.2.2.

The optimal timescale ordering for the development of a turbulent cascade and an efficient turbulent dissipation has been studied by Dmitruk & Matthaeus (2003) in a static layer with vanishing reflection at the open boundaries (in a domain of limited extent). It follows that the following inequalities favor the turbulent dissipation:

$$\tau_{nl}(k_n) < \tau_0 < \tau_{ref} < \tau_{cr} < \tau_f < \tau_d$$

(4.13)

The first two terms refer to the nonlinear dynamics of the system, which should be dominant on a wide range of $k_n$ if turbulence must develops. Reflection behaves as a forcing for turbulence at the large injection scales, to be effective its timescale should be short enough to act on the waves before they leave the domain. However, as shown in the linear analysis for the stationary case, $\tau_{ref}$ is not a dynamical timescale, the importance of reflection being controlled by the ratio of the wave period and the timescale itself. The requirement $\tau_{ref} < \tau_f$ must strictly hold for a monochromatic forcing; if one considers a broad band forcing given by a time-correlated function, as in our case, the relation involves both the wave period and the correlation time and does not need to hold strictly, since low frequency waves are injected even for small $\tau_f$. $\tau_{ref} < \tau_f$ as will be discussed below in sec. 4.2.2. Finally $\tau_d$ refers to scale bigger than the dissipative scale, since dissipation must be achieved at high wavenumber.

### 4.2.2 Chromospheric Turbulence

The velocity fluctuation patterns observed as granular motions on the solar disc are thought to be the main source of production of Alfvén waves. The magnetic field lines emerging from the underlying convection zone are continuously displaced, driven by the velocity fluctuations (high plasma $\beta$), and Alfvén waves are launched. However the granular motions are observed at low latitudes and it is not clear if in polar coronal holes the driving for Alfvénic
fluctuations is to be found in such photospheric dynamics (of difficult detection) or if Alfvén waves are generated deeper in the open flux tube structure.

We then consider two kinds of forcing, one in the velocity field and the other in the outward wave, to reproduce these two boundary conditions. Their schematic representation is given in fig. 4.3. The case [a] illustrates the boundary conditions for the velocity forcing. At the forcing shells a velocity field fluctuation is imposed, mimicking the granular motions at photospheric level, this is equivalent to impose a net energy flux at the base (an outward and inward flux). In the remaining shells, \( u \) is set to zero, waves are subjected to total reflection: \( |Z^+| = |Z^-| \).

The case [b] represents a situation in which the outgoing propagating wave, coming from the underlying layer, is transmitted to the chromosphere only in a small window of available wavenumbers, the forcing shells, while at the other scales no outgoing modes are present. This situation can be imputed to the transmission property of the underlying layer or to the fact that outward propagating waves have a characteristic perpendicular length scale given by the forcing shells, that are both quite arbitrary assumptions in absence of observational constraints. The inward propagating waves at the bottom boundary are determined by the evolution of the system and are freely propagating out of the domain at all scales.

The boundary conditions at the top [c] are not representative of a real chromosphere since no ingoing wave is present there. Instead, if a transition region is to be considered at the top of the chromosphere one has almost total reflection and hence a copious production of ingoing modes. However, in our modeling...
4.2. Turbulence in Open Magnetic Structures

Figure 4.4: Total energy in the simulation domain ($\epsilon$), net outgoing flux ($\phi$) and normalized cross helicity ($\sigma_c$) as a function of time (from top to bottom respectively), for a plane wave forcing applied to $z^+$ at the base. Time is in unit of crossing time, $\epsilon$ and $\phi$ are in arbitrary units.

the rapid flux tube expansion (see fig. 3.11) produces a strong reflection rate in the upper part of the domain, somehow taking into account this effect.

Linear Behavior

Consider first the linear behavior of the system when a plane wave of a given (low) frequency is applied as a forcing on the outgoing wave (so just in the forcing shells in fig. 4.3 case [a]). The crossing time is $\tau_{cr} \approx 500$ s, the forcing time is $\tau_f \approx 1000$ s while the minimum reflection timescale is $\tau_{ref} \approx 4$ s (cfr. fig. 4.5 for a comparison of these timescales). As the waves propagate outward their amplitude grows (WKB effect) but also reflection increases (because of the density gradient). For low enough frequency waves, the reflection timescale is comparable to the Alfvén timescale and a large amplitude backward propagating wave is generated at the top of the domain. This one, in turn, propagates backward and is partially reflected at the bottom of the domain. The continuous (strong) reflection at the top of the layer yields a characteristic timescale associated to the response of the system, namely two crossing timescales, however open boundary conditions prevent any kind of resonant behavior (as observed for loop like boundary conditions) and when a stationary forcing is applied they relate this characteristic timescale to the relaxation time of the system to steadiness. In fig. 4.4 the energy contained in the simulation domain, the net flux escaping from the boundaries and the normalized cross helicity are plotted as a function of time. The energy is mainly given by
the amplitudes of the fields in the bottom part of the domain because the density decreases exponentially and the field amplitudes increase, the geometrical expansion compensates only partially (a factor of $50 \times 150$ against seven orders of magnitude). The net flux is sensible to the dynamics at the borders of the domain, hence to what happens at the top boundary also, where reflection is higher. In the first crossing time only outgoing waves contribute to the energy level inside the box ($\sigma_e \approx 1$). After $t = 1$ forward propagating waves leave the internal domain from the top (the first spike in the net flux), ingoing waves are copiously produced, since reflection is higher at the top of the domain, and the energy in the box still increases. After $t = 2$ the backward propagating waves reach the bottom boundary and begin escaping from the internal domain. The system, which was basically accumulating energy through reflection, begins to get rid of the injected energy. When the level of cross helicity is approximately constant ($\tau \approx 3$), the energy content is regulated through the flux at the boundaries. The net flux becomes positive (energy escaping form the domain) and an approximate stationary state is reached in the following 3 crossing times (steadiness is achieved after $t \approx 12$). Depending on the frequency of the forcing the relaxation time varies: for lower frequencies reflection is more efficient and the accumulation phase lasts longer, conversely for higher frequencies the flux dominated regime becomes soon dominant and steadiness is readily achieved. Note that the velocity fluctuation at the bottom boundary is determined by the system itself which adjusts to the forcing through the energy flux.

Consider now the same plane wave forcing applied to the velocity fluctuations, $u$, at the base, that is a linear combination of ingoing and outgoing wave $u_f = 1/2(z^+_f + z^-_f)$, where the subscript $f$ denotes forcing variables. Initially no ingoing waves are generated by the reflection and we simply force on $z^+$. However, later on, the large amplitude reflected waves at the top of the domain propagate back to the base and the response of the system determines the value of the outgoing waves through the relation $z^+ = 2u_f - z^-$, consequently the energy flux through the bottom boundary is given by $\phi \sim |z^+|^2 - |z^-|^2 \sim u^2_f - \Re[z^-u^*_f]$. Depending on the relative phase of $u$ and $z^-$ and on the amplitude of the reflected wave the flux at the base injects or removes energy from the system. This is equivalent to assign a level of cross helicity at the base which is not necessarily the one the system naturally adjusts to. The relaxation time is hence longer and the oscillations in the energy reflect the oscillations of the net flux at the base.

The comparison between the propagation and reflection coefficients, as defined in eq. 2.38, is plotted for 3 representative frequencies ($\tau^* = 100, 1000, 5000$ s) in the top panel of fig. 4.5, while in the bottom panel the corresponding total energy is plotted as a function of time. For high frequency forcing, the produced ingoing waves have small amplitudes and oscillations are negligible, decreasing the frequency increases the amount of reflected waves, which are not simply produced at the top of the domain (cfr. top panel in fig. 4.5), and
4.2. Turbulence in Open Magnetic Structures

Figure 4.5: Bottom panel. Stack plot of the total energy contained in the simulation box as a function of time when a plane wave forcing is applied to the velocity fluctuation at the base. From top to bottom the frequency of the forcing is increasing, $\tau^* = 100$ s, 1000 s, 5000 s, time is in unit of crossing time. Top panel. Comparison of the reflection timescale ($|V'_{a}| = 1/\tau_{ref}$) and the forcing frequency ($\omega = 2\pi/\tau^*$, dotted line) for the chromospheric layer. In the bottom part of the domain the flux tube expansion reduces reflection, while at the top the density exponential fall off causes the increase of reflection.

the relaxation time becomes longer and longer. The forcing on the velocity fluctuations imposes a time dependence of the net flux at the base which is not the same given by the response of the system and oscillations in the energy level arise.

When the time-correlated forcing is applied the level of normalized cross helicity is always very low (about 0.2) independently of the value of the correlation timescale ($100 \text{ s} < \tau^* < 5000 \text{ s}$). In fact, the identification of low or high frequencies depends on the atmosphere chosen, in our case (chromospheric layer) $\tau^* = 100$ s approximately fixes the boundary between the two (cfr. fig. 3.19 for the transmission coefficient of the assigned atmosphere).

Nonlinear Behavior

Energy Balance Nonlinear terms conserve the total energy in a plane, the energy density at a given position $r$ satisfies the equation:

$$\frac{\partial \epsilon_w}{\partial t} = -\nabla \cdot \phi_w - Q.$$  (4.14)
Consider first the simulation R2 (forcing) in fig. 4.6 the energy, the net flux (also decomposed in flux at top and bottom boundary) and the dissipation are plotted as a function of time. Their time averaged values, in normalized units, are \( \langle \epsilon_w \rangle = 0.16, \langle D \rangle = 0.05, \langle \phi \rangle = -0.09 \). The net flux has been rescaled by a factor 10 in figure and it is the dominant term in eq. 4.15, moreover \( |\phi_{\text{bot}}| \approx 10 |\phi_{\text{top}}| \) hence the flux at the bottom basically determines the energy level. The dissipation and the flux at the top boundary become non negligible only after \( t \approx 3 - 4 \cdot \tau_{cr} \) are necessary to fill the simulation volume at the forcing wavenumbers, soon after the nonlinear dynamics develops and turbulent dissipation is reached in the next 2 crossing times. The energy injection at the bottom boundary varies on timescales longer than the forcing and crossing timescale, and triggers the dissipation events. Their delay is generally less than one \( \tau_{cr} \), indicating that the cascade time is smaller than (but comparable) to the linear crossing time, and usually they last longer than the energy injection phase (negative \( \phi_{\text{bot}} \)), suggesting that energy is trapped in the simulation domain.

Dissipation depends on the efficiency of the turbulent cascade in transferring energy to small scales, this in turn increases when \( Z^+ \approx Z^- \) while it decreases if
one of the two species dominates. The boundary condition \( u = 0 \) on the non-forcing shells has to main consequences. From one side it allows the reflected energy to be channeled in the magnetic fluctuations so that at the bottom \( E_b > E_u \). On the the other side it means that no energy is injected or is escaping from the bottom, enforcing \( Z^+ \approx Z^- \) (total reflection). The energy injected by the forcing is partially reflected at the top of the domain by the strong density gradients and totally reflected at the bottom in the non-forcing shells. The wavepackets travel back and forth partially losing their energy at the top boundary and partially transferring it to the smallest scales at the bottom of the domain, triggering dissipation which last for typically 2-3 crossing times. It follows that most of the dissipation occurs in the bottom part of the domain where \( E_b >> E_u \) and since the energy cascades to small scales just here, also \( D_b > D_u \)

Consider now the case of \( z \) forcing (run R1), whose energy, flux and dissipation are plotted in fig. 4.7. The net flux oscillates with a characteristic timescale given by \( 1/2\tau_f \) (in this case it is also approximately \( \tau_{cr} \)) and so the energy does; the dissipation events are well correlated with \( \phi_{top} \), with eventually a short delay. Bottom boundary conditions now imposes \( Z^+ = 0 \) at the non-forcing shells. Again it produces two consequences. Downward propagating waves are free to leave the simulation domain from below and equipartition between the kinetic and magnetic energy is enforced in the non-forcing shells. In this case the dissipation is regulated by the reflection mechanism at the top of the domain. When a large outward propagating wave reaches the top boundary, reflection causes a large amplitude reflected component which travels back and interacts nonlinearly, transferring energy to smaller scales, but it leaves the simulation
4. Modelling Nonlinear Interactions

Figure 4.8: Timescale ordering for a typical simulation setup in a static layer (see table 4.2) for the outward and inward propagating waves at the middle of the domain. The timescales are normalized to the crossing time (dotted line), the nonlinear timescales, $\tau_{nl}^{\pm} = (k_n Z_n^\pm)^{-1}$, are plotted in solid line, the circles mark the shells on $\tau_{nl}^+$ and the thickened line indicates the forcing shells on $\tau_{nl}^-$. Domain as it reaches the bottom boundary (hence in a crossing time). The energy cascade develops far from the boundaries, since one of the two species must vanish there (either $Z^+$ at the bottom or $Z^-$ at the top) and most of the dissipation occurs in the central part of the domain. Despite the time averaged energy $\langle \epsilon_w \rangle = 0.16$ is approximately the same of run R2, the averaged dissipation and net flux are about the half and less than half respectively, since no energy is trapped and later dissipated (see tab. 4.3 at the end of the chapter).

**Timescale Analysis**  In fig. 4.8 the time averaged timescales entering eq. 4.1 (normalized to the crossing timescale) are plotted as function of the perpendicular wavenumber for different altitudes (bottom, $\approx 1/4$, $\approx 3/4$ and top of the domain) for R2. As a general feature, both $\tau_{nl}^+$ and $\tau_{nl}^-$ decrease with distance at almost all scales. The flux tube expansion increases the nonlinear timescales ($k_l$ decreases with $r$), however, reflection causes the growth of the wave amplitudes with the radial distance (see below for the rms field amplitudes) which overcomes the freezing effects of expansion, finally reducing the nonlinear timescales. The effect of boundary conditions is clearly visible in the first and last panels. At the bottom $Z^+ \approx Z^+$ at all scales, except at the forcing shells for which $u \neq 0$. At the top boundary instead $\tau_{nl}^+ >> \tau_{nl}^-$: despite $\tau_{ref}$ is very small, the effect of reflection (equal $Z^+$ and $Z^-$) is somewhat suppressed since $Z_n^-$ must vanish there.

The smallest timescale is $\tau_{al}$, the Elsässer fields at a given position change mainly because of their propagation to the neighbouring planes. $\tau_{al}$ decreases with distance but also $\tau_{nl}$ does: two crossing wavepackets interact for a longer
time but their interaction is weaker in the bottom part of the domain, while
the interaction lasts shorter and is stronger at the top.

The only other timescale which can be smaller then the nonlinear timescale is
\( \tau_{ref} \), which varies with distance and can be used to distinguish between low and
high (perpendicular) wavenumbers: for low \( k_{\perp} \), \( \tau_{nl} > \tau_{ref} \), while the opposite is
true for high \( k_{\perp} \): reflection determines the solutions of the former, while for the
latter nonlinearities dominate. Finally note that the dissipation coefficients are
constants so that the dissipative length scale defined as \( k_{d} = \frac{\tau_{nl}(k_{n})}{\tau_{nl}(k_{n})} \),
moves to higher wavenumber as \( r \) increases, consequently the inertial range
expands to higher wavenumbers with increasing \( r \).

**Dissipation and rms Fields** Since both \( \epsilon_{w} \) and \( D \) are computed integrating on the volume, it is useful to study the rms of the fields (fig. 4.9) and
their dissipation (fig. 4.10) along the radial direction. We anticipate that the
perpendicular energy spectrum decreases with wavenumber at any position,
therefore the quantities derived integrating over the shells, such as the rms
value of the fields, are mainly given by the solutions at low wavenumbers. At
such scales the relation \( \tau_{nl} \gg \tau_{cr} > \tau_{ref} \) approximately holds in the entire
domain, it follows that the rms amplitudes are approximately given by the
solutions of the linearized eqs. 4.1 for low frequency waves.

In the upper domain the kinetic fluctuations are larger than the magnetic
ones, while in the bottom part, say for \( r \lesssim 1.001 \) boundary conditions deter-
mine the relative amplitude between kinetic and magnetic fluctuations. For
the \( z \) forcing case, as the radial distance increases, soon happens that \( |u| > |b| \),
so that the total energy is kinetically dominated. For the \( u \) forcing case instead boundary conditions allows to store energy in the magnetic field and the
condition \( |u| > |b| \) is reached at almost the middle of the domain. In the region
of high density the magnetic fluctuations are bigger than the kinetic ones and
equipartition in the volume integrated energy level holds, the fields at bottom
compensate for the imbalance in the upper part of the domain.

The rms Elsässer fields are similar for both the kind of forcing (right panels
in fig. 4.9), since bottom boundary conditions affect the field amplitudes only
at high wavenumbers. Both WKB effects and reflection make the amplitude
to increase with distance. This can be used as a diagnostic tool for dissipa-
tion. Contrary to the energy \( \epsilon_{w} \), the energy per unit mass, \( E_{tot} = E^{+} + E^{-} = \frac{1}{4}(|Z^{+}|^{2} + |Z^{-}|^{2}) \), is mainly given by the fields at the top of the domain. Since
\( Z^{+} \gg Z^{-} \) there, it follows that the temporal variation of the flux at the top
reflects the variation of \( E_{tot} \). In the bottom panel of fig. 4.7 it is evident a clear correlation between dissipation and the energy per unit mass. A dissipation
events is delayed less than a crossing time and lasts for approximately a
crossing time, indicating that dissipation is triggered by reflection at the top
boundary and that the reflected energy escapes from the bottom. For the \( u \)
forcing (bottom panel of fig. 4.6) a clear correlation is not established, of course
the above dynamics is still working, but sometimes dissipation last longer than a crossing time after the flux injection enhancement, supporting the fact that accumulation of energy occurs and dissipation is stronger in the bottom part of the domain.

The dissipation is dominated by the magnetic one, whatever the boundary conditions are and in the whole domain (left panels in fig. 4.10). However, some differences still exist, mainly due to the boundary conditions, which influence the cascade of the dissipative scales. For example, in R1, $z^+ = 0$ at the non-forcing shells, the dissipation in the bottom part of the domain is given by the damping of the inward propagating waves. On the contrary for R2 we impose an equal amount of inward and outward waves and both modes contribute to dissipation. As a result the total dissipation is about twice bigger. Note, however, that also in this case the inward modes contribute more than the outward modes in the bottom part of the domain. In a similar way boundary conditions affect the relative damping between magnetic and velocity fluctuations, but no clear imbalance can be established here, except that mentioned above. The dissipation per unit mass, which enters directly in eq. 4.1, follows strictly the wave amplitudes, being much higher at the top boundary. Here $z^- = 0$ maintains $Q^+ / \rho \gg Q^- / \rho$ and $Q_{\mathrm{u}} / \rho = Q_{\mathrm{b}} / \rho$. However as we move further in, the magnetic dissipation dominates and the imbalance between the inward
4.2. Turbulence in Open Magnetic Structures

Figure 4.10: Upper panels. Time averaged dissipation \( \langle Q \rangle \) as a function of radial distance (solid line) for R1 in normalized unit. On the left, the contribution of the magnetic and kinetic dissipation \( Q_b \) and \( Q_u \) (dotted and dashed lines respectively) is shown, while on the right the contribution of the ingoing and outgoing mode are plotted (dashed and dotted line). Lower panels. Same plot for run R2.

and outward contribution decreases (as show for \( Q \)). Again for the \( u \) forcing case the dissipation per unit mass is about twice than that of the \( z \) forcing case: therefore the total reflection at the bottom boundary allow the energy to cascade in the whole domain and not only at the bottom, as might be deduced by the integrated dissipation \( D \).

Scale by Scale Energy Budget The energy residing in the small scale fields, and hence dissipation, is determined by the efficiency of nonlinear interactions in transferring energy to the high wavenumbers. Phenomelogical arguments (sec. 2.3.3) tell us that whatever the imbalance between the inward and outward modes is, if an inertial range is established the energy is transmitted at the smaller scales at the same rate maintaining \( E^+E^- \propto k^{-4} \) but nothing can be said about how the imbalance depends on the wavenumber. In our system, boundary conditions on the non-forcing shells impose some limits on the ratio \( E^+/E^- \) and the cascade may proceed asymmetrically. Moreover, the forcing is applied on the outgoing mode only and at a given position, the variation of the energy is subjected to propagation effects, reflection effects and finally to nonlinear interactions. The first is the same independently of the wavenumber and leads to the decorrelation effect (introduced in sec. 2.3.1). The timescale analysis discussed above shows that reflection can dominate on
nonlinearities at certain scales as well as the contrary may happen. Assuming a stationary state, the variation with position of the energy contained in a given shell, $E^\pm_n$, may be written as:

$$\pm V_a \frac{\partial E^\pm_n}{\partial r} = \pm \left( \frac{\partial}{\partial A} + \frac{\partial}{\partial V} \right) V_a E^\pm_n \mp \frac{V'_a}{V_a} V_a E^\pm_n - \omega^\pm_n + f^\pm_n - \pi^\pm_n$$  \hspace{1cm} (4.16)$$

where $E'_n = u^2_n - b^2_n$ is the energy difference and $\omega^\pm_n = \nu k^2 E^\pm_n$. On the RHS, from left to right, appear the two reflection terms (WKB and mixing term respectively) which depend on the radial variation of the background equilibrium field. The remaining terms are the dissipation term, $\omega^\pm$, which still depends on the radial direction through the wavenumber $k(r) = k(r_b)/\sqrt{A}$ ($r_b$ is the bottom boundary), but also on the efficiency of the turbulent cascade; the forcing term, $f^\pm$, which is applied only at large scales ($n_f = 1, 2, 3$) at the bottom boundary; and finally the energy flux due to nonlinear interactions, $\pi^\pm_n$ (its expression in given by eq. B.2 in appendix, changed of signed). Since $\tau_{el}$ is the smallest timescale, the forcing applied in the first plane is still acting at the large scales also is the other planes, advected by propagation, and one can assume that $f^+_n \approx f^+_n$ for $n = n_f$ at all position and $f^-_n = 0$, $f^+_n = 0$ elsewhere.

For all correlation times considered the magnitude of the linear terms in eq. 4.16 is larger than that of the nonlinear flux: turbulence may be considered weak, in the sense that it does not affect the energy level of large scales, though some energy is transfered to the the smaller scales. From a “cascade point of view”, in each shell the WKB and mixing terms act as a forcing, defining the overall level of energy in the two species, and hence influence the energy flux through the product $E^+ E^-$. The WKB terms account for an increase of the fields, separately, in the radial direction (although boundary conditions limit this effect for the $z^-$ to the middle-lower domain), they are always positive (the term in brackets is $1/(2H)$, where $H$ is the density scale height) and do
not alter the ratio of the two modes, so that if an imbalance exists at \( r = r_b \), it is maintained constant throughout the domain; the mixing term, instead, redistributes the energy among the two species (\( z^+ \)) in the radial direction and its sign depends on the sign of the Alfvén speed gradient and on that of the energy difference. The former is negative below \( R \approx 1.0007 \, R_\odot \) and becomes positive above this point; its magnitude is greater in the upper domain, where we expect the mixing term to be more important.

In fig. 4.11, the residual energy (\( \sigma_d \), right panel) and the normalized cross helicity (\( \sigma_c \), left panel) are plotted as a function of the radial distance and the shell number. At \( R \approx 1.0007 \, R_\odot \), \( V'_u = 0 \) and the mixing term vanishes so that the coupling among the two species is given only by the nonlinear interactions and one can assume that the cascade proceeds as in the homogeneous case, with the WKB terms acting as a forcing: at this position \( \sigma_d \approx -0.3 \) at almost all scales.

In the upper part of the domain, boundary condition force \( E^+ \gg E^- \), the energy flux is hence reduced with respect to its “naturally” symmetric case, the amplitudes of the fields are given by the linear terms in eq. 4.16 and the inertial range is confined to very high wavenumbers. At the bottom, below \( R \approx 1.0007 \, R_\odot \), boundary conditions force \( E^+ \approx E^- \) in the \( u \) forcing case (the one plotted in the figure) and the flux is increased with respect to its “naturally” less symmetric expression. However, since fluctuations have low frequencies, the contribution of the linear term is not negligible. In this region, the residual energy is negative and close to -1, because the imposition \( u = 0 \), and \( V'_u < 0 \). The mixing term adds energy to the ingoing mode, as a result \( E^+ \lesssim E^- \) at the bottom and accordingly \( \sigma_c \lesssim 0 \). Ultimately the imbalance in the dissipation is caused by the imbalance of the energy, which in turn is caused by the linear terms in eq. 4.16.

The value \( \sigma_d \approx -0.3 \), found at the location of the vanishing Alfvén speed gradient, can be used as an indicator of dominant nonlinear coupling (with respect to the linear one), i.e. of enhanced nonlinear interactions. Comparing fig. 4.8 and the right panel of fig. 4.11 one can identify the inertial range (\( \tau_{nl} < \tau_{ref} \)) with the range of shells for which \( \sigma_d \approx -0.3 \), while at the dissipative range \( \sigma_d \lesssim -0.6 \).

**Perpendicular Wavenumber Spectra** The spectra in the perpendicular wavenumber are determined by the relative magnitude of the energy fluxes and the linear terms in eq. 4.16. Generally they will differ depending of the location of the plane: the relative magnitude of the energy flux increases with distance as long as the mixing term maintains a small imbalance among the fields, which, however, is dramatically altered by the boundary conditions. In fig. 4.12 the spectra for the elssser fields (left panel) and the velocity and magnetic fluctuations (right panel) are plotted at four different positions for run R18. For the \( z \) forcing case the same plots are obtained, except that the
signature of the forcing appearing at low wavenumber at the bottom is in the $z^+$ spectrum instead that in the $u$ spectrum.

In the upper part of the domain boundary conditions reduce the power residing in the inward mode which is about two orders of magnitude smaller the the power of the outward mode, while elsewhere the power is comparable with a weak dominance of the outward mode. The flattening of the spectrum at $r \approx 1.0031 \, R_\odot$ is due to the linear behavior which dominate at small wavenumbers. Instead, at large wavenumbers nonlinear terms are efficient in shaping the spectrum and one can follow the evolution of the inertial range. It forms at about $\log_{10} k = 4$ close to the base and moves to higher wavenumbers with increasing distance (compare the contour of $\sigma_d$ in fig. 4.11). Note that indicating with $m^\pm$ the slopes of the spectra in the inertial range, the rule $m^+ + m^- = 4$, is approximately satisfied, even if it could be a fortuitous coincidence.

4.2.3 Heating and Evolution of Turbulence in the Solar Wind

The Alfvén waves observed in the corona (at heights above $3 \, R_\odot$) can be thought as part of the wave flux which is trasmitted through the transition region (therefore the remnant of the photospheric flux); alternatively the observed flux could be produced by the shear interaction among plumes and interplumes in the expanding coronal hole, or other local porcesses (e.g. magnetic reconnecction) which occur in the low corona. We will consider here only the first case, as a natural outcome of the dynamics described in the previous section. The base of our modeled corona is located just above the transition region and the waves are injected directly at the base ($z$ forcing case) in a limited number of shells (there is no reason for considering velocity patterns at coronal heights). The value of the lowest wavenumber is set in order to match, at the forcing
shells \((n_f = 1, 2, 3)\), the characteristic length scale of the photospheric (super-meso)granular motions, which are larger at coronal heights because of the flux tube expansion in the chromosphere. Boundary conditions on the non-forcing shells at the bottom boundary are not truly representative. In fact, we completely neglect the outgoing waves entering the corona at high perpendicular wavenumbers (as happened in the chromosphere for the same forcing, \([b]\)) for whose absence, in principle, there is no reason: in the very low corona the turbulent cascade is hence “minimal” because of the lack on outgoing mode at high wavenumber. The flux of outgoing waves at the base of the model is not constrained by observation, while measurements of non-thermal width of the emission lines give an upper limit to the velocity fluctuations. Accordingly, for all the run considered (tab. 4.2) we adjust the input flux in order to obtain a rms value of the velocity fluctuations \(\delta u_0 \approx 33 \text{ km s}^{-1}\). At the upper boundary waves are allowed to freely escape from the internal domain since both the mode are propagating outward, avoiding the problems encounter in the chromosphere.

**Reflection and Nonlinear Interactions**

The modeled corona is the same of that one described in sec. 3.2.1 for the semi-analytical nonlinear model (see fig. 3.11). The reflection coefficient is high in the low corona, say below \(5 - 6 R_\odot\), so that most of reflection occurs in this limited layer, unless waves have very low frequencies. The Alfvénic critical point is at about \(13 R_\odot\) and beyond this point waves are advected outward by the solar wind: only wave reflected inside \(X_a\) can propagate back to the base.

This behavior is quite evident in the left panel of fig. 4.13 in which the normalized energy of the inward wave is plotted as a function of distance and time. The systematic growth of the wave amplitudes, due to the variation of the phase speed, is removed through a normalization, which consists in dividing the energy at a position \(r\) and time \(t\) by the time averaged value of the energy at that position \(\langle E^- (r)\rangle_t\). The mechanism of reflection around the Alfvénic critical point produces an accumulation of ingoing modes, \(z^\dagger\), which at that location have a vanishing phase speed; this part of the reflected energy propagates forward and backward above and below \(X_a\) respectively. The propagation path is almost horizontal and it bends as we move toward the base, becoming almost vertical \((V_a \approx 3000 \text{ km s}^{-1})\) or toward the outer domain (the bend is less significant since \(|V_a| \approx 1/2U\) at \(25 R_\odot\)). At large distance the phase speed of both the species is given mainly by the wind speed and the propagation patterns assume almost the same slope (still at \(50 R_\odot\), \(|V_a| = 1/4U\) and the slopes are not the same at the top of our domain). It is interesting to note how reflection works in producing the backward propagating waves. In the figure (left panels) two populations of \(z^\dagger\) appears. The first is that one which accumulates at \(X_a\) and propagate slowly outward and inward. The other, instead, is continuously generated by the reflection
4. Modelling Nonlinear Interactions

Figure 4.13: Normalized Elsässer energy of the ingoing mode as function of time and position (in unit of crossing time and solar radii respectively) for run W6 (left panel) and run W2 (right panel). Since the energy increases with distance, in each plane it has been normalized with respect to its time averaged value in order to highlight the propagation pattern. Black lines trace some local maxima of the normalized Elsässer energy $E^+_N$ of the outgoing mode (whose local maxima are indicated with black lines in the figure) and propagates with its same phase speed, hence outward (left and right panels). It is the equivalent, in the non-WKB case, of the secondary, or anomalous, component introduced in sec. 2.3.3. When this two populations meet, a high level of $E^-_N$ appears on the plot. As long as the outgoing waves have high frequencies the anomalous component has a small amplitude with respect to the outgoing mode, in almost all the domain and is similar to the WKB case; for low frequency waves, reflection is much higher in the lower corona (say below $X_a$) and the secondary, anomalous, reflected wave is no more negligible compared to its mother outgoing wave, and its behavior departs form the WKB case (right panel).

**Averaged Profiles along the Radial Direction**

Since reflection is on average lower than in the chromosphere and the wave amplitude are larger, the development of the turbulent cascade and hence the efficiency of dissipation depends critically on the frequency of the injected waves. Recall that the correlation time control the duration and the period of the forcing wave: for low $\tau^*$, short living, high frequency wavepackets are injected in the corona and the resulting power spectrum is horizontal (uncorrelated low-frequency fluctuations) for $\nu < \nu^* = 1/(2\tau^*)$, peaks at $\nu^*$ and decreases rapidly above it.

In fig. 4.14 the time averaged amplitudes and the dissipation per unit mass are plotted as a function of radial distance for six runs with different forcing
timescales. Very high correlation times (the two upper panels) produce almost an equal amount of ingoing and outgoing fluctuations, the \( z^+ \) has a maximum at the Alfvénic critical point, as predicted by the linear theory, and dissipation is peaked very close to the base (at about \( 2 R_\odot \)) and made up of equal contributions from the two species. Decreasing the correlation time, decreases dramatically the level of ingoing modes (less reflection), but dissipation is enhanced to a level four times bigger. It also seems to reach an asymptotic profile with a peak shifting from higher altitudes toward \( 4 R_\odot \), and becoming more sharp.

These differences arise from the kind of forcing which is applied and the mechanism of reflection. For large \( \tau^* \), we are injecting very low frequency waves which are coherent on several nonlinear timescales. Reflection is very efficient in producing the ingoing modes which are continuously propagating backward from above. The majority of reflected waves are created at a distance much lower then \( X_a \), say below \( 8 R_\odot \), and their propagation time is hence \( \lesssim 10000 \) s (see fig. 4.13). When two wavepackets interact, the counter-propagating fields have almost opposite phases, therefore the resulting variation of amplitude due to nonlinear interaction is negligible, i.e. the turbulent cascade is suppressed.

Consider instead the opposite case in which high frequency, short living wavepackets, but also low frequency, uncorrelated fluctuations, are injected at the base. Now reflection works at a smaller rate in producing the ingoing waves, which are a small fraction of the outgoing mode, but interactions occur among uncorrelated wavepackets and are more efficient in transferring energy to the dissipative scales. The turbulent cascade proceeds asymmetrically, since \( z^+ \gg z^- \) at all scales, and the enhanced level of dissipation is due to the damping of the outgoing mode sustained by a small level of ingoing modes. The intermediate situation is represented by the central panel in fig. 4.14, for which \( \tau^* = 10000 \) s, that can be consider as the characteristic time (for this model atmosphere) separating the two regimes (\( \tau_s \)).

For long correlation times, the peak of dissipation is close to base since the reflected waves when, arrives there, have had enough time to change their phase. Decreasing the correlation time, waves can interact efficiently at higher and higher distance, the amplitude of the fluctuations grows with distance (WKB effect) and the dissipation (per unit mass) increases. It is intersting to note that the anomalous reflection is able to provide a non-negligible amount of dissipation in the extended solar wind, accounting for the decrease of the Elsässer fields, almost in accordance with observational constraints given by Helios and Ulysses measurements as shown in fig. 2.4. Using the estimated slopes for the radial decrease of the fields to extrapolate back to \( 50 R_\odot \), one obtains values for the field amplitudes, \( z^\pm \), of about \( 230 \) km s\(^{-1}\) and \( 50 \) km s\(^{-1}\) respectively, not far from those obtained with the highest correlation time (\( 300 \) km s\(^{-1}\) and \( 90 \) km s\(^{-1}\) respectively).
4. Modelling Nonlinear Interactions

Figure 4.14: Time averaged amplitudes of the Elsässer fields ($|z^\pm|$), solid and dashed lines respectively, left panels) and corresponding dissipation per unit mass ($Q/\rho$ in normalized unit, solid line, right panels) as function of distance for different runs. The contribution of the outgoing and ingoing modes to the dissipation is plotted in dashed and dotted line respectively in the right panels. For all runs $\delta u(R = R_\odot) \approx 35 \text{ km s}^{-1}$, while the correlation time of the forcing, $\tau^*$, varies as indicated in figure.

Perpendicular Wavenumber and Frequency Spectra

We have seen that a turbulent dynamics develops in the perpendicular plane when low frequency uncorrelated waves and high frequency waves interact nonlinearly, therefore turbulent spectra are found only for $\tau^* \lesssim \tau_s$. In this condition the outward mode still dominates at all scales ($\sigma_c \lesssim 1$), while there is a weak imbalance between the magnetic and kinetic energy ($\sigma_d \lesssim 0$). The energy flux is approximately constant over a wide range of scales (two or three decades) and $\Pi^+ >> \Pi^-$ in the whole domain. As can be seen in the top panels of fig. 4.15, an inertial range develops at just two solar radii. The slopes of the spectra of the two modes are very different at the base, the outgoing energy spectrum being much steeper, probably because of the boundary conditions. At greater distance the spectra slightly flattens in the inertial range and approach the same asymptotic slope, close to the Kolmogorov 5/3. At low wavenumber the expansion effects freeze the spectral evolution for the outgo-
4.2. Turbulence in Open Magnetic Structures

Figure 4.15: Top panels. Perpendicular wavenumber spectra $\epsilon^\pm = \rho E^\pm$ (left and right panel respectively) for run W6 at different position. Distance is increasing from top to bottom: $R/R_\odot \approx 2, 7, 13, 25, 50$. Values are in normalized unit. The power law scalings, with the slopes indicated in figure, are plotted for reference. Bottom panels. Smoothed frequency spectra of the Elsässer energies, $E^\pm$, for the same run W6 at different position, $R/R_\odot \approx 2, 7, 13, 50$, in black, blue, light blue and red line respectively. In each panel values are in normalized units.

The frequency spectra (bottom panels) do not evolve much with distance (except for the overall level which scales according to fig. 4.14). The signature of the forcing, at the frequency $\nu^* = 1/(2\tau^*)$, persists in the outgoing spectrum, even at large distances. In the ingoing spectrum a small forcing peak appears immediately close to the base and disappears only at large distances. This behavior is related to reflection which transfers part of the outgoing energy to the ingoing modes mainly in the inner corona, and hence the appearance of the forcing is related to the persistence of the forcing in $E^+$. The shape of $E^+(\nu)$ close to the base is given by the power spectrum of the forcing and evolves only at frequencies greater than $\nu^*$: the high frequency branch can be associated to the nonlinear interactions, whose characteristic timescale decreases with distance approaching the propagation timescale (the smallest timescale).
Below the Alfvénic critical point nonlinear interactions transfer energy to small scales in the perpendicular wavenumbers (flattening of the $k_\perp$-spectra) and also the high frequency branch of the spectrum increases its energy content. Above $X_a$, nonlinear interactions (and dissipation) are reduced, the triggering coming only from the anomalous reflection, so that the shape of the spectrum is mainly given by the WKB effects (the spectrum is less noisy). The ingoing spectrum decreases monotonically with an approximate power law scaling of slope $-2.75$. Again the variation of the slope at high frequencies is related to the power residing in the fluctuations at high wavenumbers, i.e. on the efficiency of the turbulent cascade in the perpendicular planes. The evolution of the $E^+$ and $E^-$ spectrum at high frequencies is approximately the same, except close to the base, where boundary conditions affect the outgoing spectrum.

### 4.3 Discussion

The efficiency of turbulent dissipation, driven by reflection of Alfvén waves in the chromosphere and corona, has been studied in relation with the problem of coronal heating and acceleration of the solar wind.

The model atmosphere, used to represent the chromosphere, presents several simplifications concerning the background equilibrium density and magnetic field configuration, and above all the temperature profile (assumed to be isothermal), although it reproduces the gross features of the chromospheric layer. The most severe limitations arise from the exclusion of the transition region, which affects the boundary condition at the top of the layer. In the corona, the problem of the topping of the atmospheric layer is circumvented extending the computational domain above the Alfvénic critical point, but the problem of bottom boundary conditions and forcing is, of course, still present, although it affects less significantly the results. Despite these limitations, some conclusions may be derived, concerning the dependence of the turbulent dissipation on the kind of forcing applied at the bottom boundary and on the related boundary conditions in photosphere. Moreover, the conditions under which reflection actually drives the turbulent cascade shows up clearly when the mechanism is studied in the corona.

For the chromosphere two kinds of forcing were applied, the first to the velocity fluctuation, mimicking the photospheric pattern of the supergranular motion, the second directly to the Alfvén waves propagating outward, in order to reproduce the effect of a flux of wave propagating form the underlying layer. Boundary conditions also varies accordingly to the forcing. For the $u$ forcing case, total reflection on the non-forcing perpendicular length scale is imposed ($u = 0$) for waves approaching the boundary from the internal domain. For the wave forcing instead a vanishing outward flux is imposed, which result in an equipartition among velocity and magnetic fluctuations ($u = b$).
In table 4.3, the time averaged energy \( \epsilon_w \), net flux \( \phi \) and dissipation \( D \) (in normalized units \( \times 10^3 \)) together with the forcing amplitudes \( (z_f, u_f \text{ in km s}^{-1}) \) and the correlation time scale \( \tau^* \) (in \( 10^3 \) s). For the Corona, the forcing is applied only on the outgoing mode, the rms values computed at the bottom boundary, \( u_{bot} \) and \( z_{bot} \), are in \( \text{km s}^{-1} \), the simulation duration, \( t_{max} \), is in crossing time (about \( 26000 \) s) and the net flux at the bottom boundary, \( \phi_{bot} \), is in normalized units as above.

In 4.3. Discussion, the time averaged energy, dissipation and net flux are listed for different runs in which the correlation time and the amplitude of the forcing are varied: \( \tau^* \) specifies the period and the correlation time of the forcing function (the correlation time is about \( \tau^* \) and the period \( \tau^*/2 \)). For a fixed amplitude of the forcing, the energy and the dissipation do not vary with the forcing timescale in a simple way. For all the parameters considered, the ratio of the dissipated energy to the energy contained in the simulation is higher for the \( u \) forcing case, as one expects noticing that total reflection at the bottom boundary traps the waves in the simulation box, which, therefore, have more time to interact nonlinearly and hence dissipate: the footpoint motion scenario is a more favorable condition for the development of turbulence than the wave-flux scenario, mainly because of the different boundary conditions. However, inspection of the table reveals that this is not the whole story.

Consider first the \( z \) forcing case, in which waves are freely escaping from the top and bottom boundaries. Increasing \( \tau^* \), the ratio of dissipation to energy increases because of the enhanced level of reflection. However, as the correlation time (and period) further increases, the above ratio decreases, but the outgoing wave flux increases (in table a negative flux signifies injection of energy): waves are escaping from the internal domain with negligible dissipation. Rescaling the amplitude of the forcing in order to get the same amount of energy in the box, the ratio between dissipation and energy decreases with increasing \( \tau^* \), contrary to the expectations since reflection is increasing.

For the \( u \) forcing case, the relative dissipation increases with the forcing timescale, both at fixed forcing amplitude or rescaling the amplitude to get the same en-
ergy content. This strange response of the system to the forcing can be understood if one takes into account that for short \( \tau^* \), short living, high frequency waves are injected, while in the opposite limit one deals with long lasting, low frequency waves. Consider the latter case. An outward propagating wave will interact with a backward propagating waves which is generated by reflection in some region of the layer and has propagated back to the base. If the correlation time is long, the backward wave is generated by the older wavefronts of the same outgoing wavepacket in the region of strong reflection. If also the frequency of the fluctuation is small, when the two waves interact, their phases will be basically in opposition and the term accounting for nonlinear interactions cancels out, suppressing the cascade of energy to the small scales, and hence suppressing the turbulent dissipation.

This explains the decrease of relative dissipation with increasing \( \tau^* \) for the \( z \) forcing case, where the above situation applies strictly. In the \( u \) forcing case, a wavepacket propagates back and forth for a long time because of the total reflection at the bottom boundary and of the high level of reflection at the top. The less the frequency of the wave the more its reflection, the longer the time it is trapped in the domain. Its phase will change and sooner or later it will interact with a counter-propagating wave which is basically uncorrelated, restoring, on average, the nonlinear cascade. As a matter of a fact, dissipation is triggered after several crossing times (about 20) when the forcing timescale is very high.

As was found with semi-analytical model, the linear terms in the evolution equation of the fields largely influence the dynamics in this highly stratified layer. The cascade develops at high perpendicular wavenumbers, dissipating the inward and outward mode at approximately the same rate, yielding \( z^+ \approx z^- \) in the inertial range (as in the usual phenomenological picture of homogeneous turbulence). At large scale the imbalance among the modes is instead determined by the reflection mechanism and boundary conditions.

The above “correlation” effect appears clearly in the results concerning the corona, for which we impose open boundary conditions at the bottom (\( z \) forcing). In this case one can also give an estimate of the maximum correlation time and period (10000 s) above which dissipation is suppressed. Note, however that low frequency-uncorrelated fluctuations are generated, even for short forcing timescale (with the form we assume for the forcing), since they are necessary to produce a sufficient amount of reflected waves, without which nonlinear interactions will be negligible. This explains also the reason for which a long lasting high frequency wavepacket would produce negligible dissipation, i.e. the stationary case considered in the semi-analytical analysis of the previous chapter (sec. 3).

From a qualitative point of view, the shape of the heating profile produced by the turbulent dissipation is in agreement with the requirement of current
turbulent driven wind models. In the low corona the cascade is already well developed and is able to transfer energy to the smallest scale. However, the peak of the dissipation per unit mass is an order of magnitude smaller than that obtained with the phenomenological model of sec. 3.3, although close to the base the heating function is higher and could increase further if more realistic boundary conditions are implemented.

In our model, the evolution of the frequency spectrum is given by the evolution of the perpendicular spectra, whose shapes, at large scales, depend also on the propagation properties of the waves: high frequency fluctuations increase moderately their energy when the turbulence is active, since nonlinear interactions develop on small timescales. The evolution of the perpendicular spectra can be explained with the same argumentation presented in sec. 2.3.3 assuming that the asymptotic equilibrium is given by a Kolmogorov spectrum in the corona (low Alfvén decorrelation effect). A critical wavenumber, in each plane, divide the perpendicular spectrum in two branches. For the lower one the adiabatic timescale is shorter than the nonlinear one and the spectrum is given by the mixing term which regulates the imbalance in the outward and inward component. For the high wavenumber branch, the spectrum evolves driven by the turbulent dynamics toward the equilibrium spectrum. The value of the critical wavenumber depend on the the radial coordinate and decreases for increasing distance. Coming back to the frequency spectrum, the peak of the forcing, applied at large scales at the bottom of the domain, persists in the whole domain, and it is not eroded by the turbulent dynamics, as expected from observations if one assumes that the Alfvénic spectrum has solar origin.
Chapter 5

Summary and Conclusions

In this thesis, we have studied the conditions under which an incompressible turbulent dynamics is able to give a substantial contribution to the coronal heating and to the related acceleration of the fast streams of the solar wind, in structures in which magnetic field lines are mainly open (i.e. in coronal holes). The turbulent motions are generated by an unidirectional flux of Alfvén waves launched from the photosphere, and develop thanks to the stratification of the background equilibrium fields (density, magnetic field and wind speed) in which waves propagate. The inhomogeneity causes the reflection of low frequency waves which can interact nonlinearly with the waves coming from the Sun. As a result, a turbulent cascade develops that transfers the energy to smaller and smaller scales, achieving dissipation of the injected energy. The models considered extend up to the outer corona and solar wind, where in situ measurements allow to test the ability of the proposed mechanism in reproducing observational constraints. These ones concern both the equilibrium fields (density, speed and temperature of the wind) and the properties of fluctuations (rms amplitudes of the outward and inward propagating waves, frequency spectra of the fluctuations as a function of the radial distance). The latters, in particular, are strictly related to the evolution and sustainment of turbulence in the solar wind, which differs from the usual homogenous and isotropic MHD turbulence, commonly investigated through full MHD simulations. Furthermore, observations have shown the existence of two contradicting features of the low frequency fluctuations, in the so called Alfvénic range: the turbulent spectrum is well developed and evolves with distance, supporting the fact that strong nonlinear interactions are at work in the solar wind (as inferred from other observational facts); on the other hand, in this frequency range, the fluctuations of the velocity field are almost incompressible and strongly correlated to the magnetic fluctuations, a property of outward propagating Alfvén waves, which, on their own, cannot undergo nonlinear interactions. Despite the weak inhomogeneity in the outer corona, reflection could represent a solution to this problem, generating the missing backward propagating fluctuations which trigger the turbulent cascade.
The work discussed relies on two different approaches to the above mechanism. First a phenomenological model for nonlinear interaction is used in order to solve the equations of Alfvén waves propagating from the photosphere to the earth orbit, damped by a turbulent-like dissipation. Despite the roughness of the description of the turbulent dissipation, this model retains a property of nonlinear interactions which is usually discarded when turbulence is invoked, and used, in solar wind models to transfer energy at small scales and achieve dissipation; namely, that nonlinear interactions occur only among counter-propagating waves.

Preliminary studies in isothermal atmospheres with wind show that lower frequency waves are subject to strong damping since their reflection is higher compared to high frequency waves. On the other hand, when the fluctuations consist of a mixture of high and low frequency waves, the slope of the frequency spectrum determines the efficiency of the turbulent dissipation. High frequency waves are poorly reflected and hence are mainly propagating outward. If their energy is relatively high, they dissipate efficiently the low frequency reflected waves, which are the driver for the dissipation in the whole spectrum. As a consequence, steep spectra dissipate more energy than flat ones.

Specifying the equilibrium fields in the chromosphere, corona and wind, this model has been applied to the solar atmosphere. Assuming a rms velocity amplitude of the fluctuation at the base of the solar corona as given by observations, and imposing a turbulent correlation length (a free parameter of the model) of the order of the dimension of supergranule at the photosphere, we are able to reproduce most of the observational constraints in the solar corona and solar wind. In particular, data of the Elsässer energies in the solar wind are better reproduced if a steep spectrum is imposed at the photosphere. However, the model is not able to reproduce the observed variation of the frequency spectrum with distance, and the above indications must be taken with some cautions. The model also fails in satisfying the observational constraints in the photosphere, so, finally, it can be considered a good approximation for turbulence in the corona but it must be refined to be successful in the chromosphere.

The encouraging results for the corona have lead to the development of a model which couples the (stationary) equations for the equilibrium fields with the equations for turbulence. This self-consistent system of equations has been solved by means of an iterative procedure in order to find a stationary solution for a turbulent driven solar wind. We have found that such a stationary solution cannot be obtained if the heating and the acceleration are supplied by the incompressible turbulence alone. The main reason for this failure is that solar wind models require a non-negligible amount of dissipation per unit mass close to the coronal base (say below $2 R_\odot$), where the high density produces the high heating (per unit volume) necessary to accelerate the wind. The phenomenological model, on the contrary, produces a large dissipation far from the base (at about $3 - 4 R_\odot$) because the wave amplitudes grow there in response of
the density exponential drop. The iteration procedure oscillates between two states, one in which the wind is accelerated far from the base, with very steep gradients before the acceleration region, and the other in which the wind is accelerated at lower heights, presenting a smoother profile. Starting from the former, the enhanced reflection at the base pushes the system toward the other solution, which in turn relaxes to the starting solution since the low level of reflection is no more able to sustain the wind acceleration. From these behavior one may conclude that other mechanisms, not relying on incompressible turbulence alone, must be invoked to obtain a supersonic solar wind with a rapid acceleration close to the coronal base (adding an ad hoc heating function close to the base, the system actually converges to a solution for the wind and the turbulence equations; this solution also satisfies the observational constraints at the base and at 1 AU). Such missing heating could be provided by compressible motions which have been neglected so far. Currently we are working on the inclusion of second order compressible effects in the turbulence equations based on the Reduced MHD equations, extended to inhomogeneous equilibrium. However the extension is not straightforward since the resulting heating presents a singularity when the fluctuations are transonic, a situation which is encountered when the iteration proceeds. Moreover, to maintain the self consistency of the model, the coupling and the resulting compressible dissipation must enter in the turbulence equations as a sink for both the counter-propagating waves, without altering the cross helicity, a condition not readily satisfied. However, without invoking extra heating, the pure incompressible model could still give interesting results (not necessarily producing solution with solar wind properties) if more physics is included in the energy equation. A possible quick fix to the problem of the oscillations between the two temporary solutions comes from the inclusion of heat conduction. In fact, it should allow to channel part of the incompressible heating to the low corona, below the temperature maximum, producing smoother profile for the density and the wind, with a stabilization effect on the runaway process described above.

If one insists on the Alfvén fluctuations as the main driver for the heating and acceleration, promising mechanisms are represented by parametric instability or other wave couplings with compressive modes. Recently, exploiting a one dimensional model, Suzuki & Inutsuka (2005) succeeded in building up a transition region and a solar wind whose properties are in good agreement with observational constraints, just injecting a flux of Alfvén waves at the base of the photosphere and allowing the coupling between the fast magneto-acoustic and Alfvén modes. The former modes steepen and form shocks in the low corona, leading to an efficient dissipation. The model, though successful in producing a supersonic solar wind, yields a low level of the temperature maximum in the corona and an extension to two dimensionality probably will weaken the resulting dissipation.
The second approach we followed is limited to the turbulence equations, hence we again neglect the back reaction of the resulting turbulent heating on the equilibrium fields, which are specified. The nonlinear interactions are treated adopting a low dimensional model for (2D) turbulence, a shell model. The dependence of the field on the scales perpendicular to the direction of wave propagation (assumed to be parallel to the mean magnetic field) is retained decomposing the corresponding 2D fourier space in concentric shells. Despite we lose spatial information on the structures forming in the perpendicular plane, we obtain a more accurate description of the nonlinear dynamics, with respect to the phenomenology adopted so far, and moreover, high Reynolds numbers can be achieved with a relatively low computational cost (compared to full MHD simulations), which allows us to consider a wide inertial range (extending for two or three decades in the wavenumbers). The solar atmosphere is decomposed in two distinct, separated, layers: the chromosphere and the corona. In this numerical simulations, the boundary conditions play a crucial role in determining the conditions for the development of turbulence and its evolution. We studied the effect of two different boundary conditions and forcing on the chromospheric turbulence. One of the driving mimics the production of Alfvén waves caused by the velocity patterns observed in the photosphere as supergranular motion, which continuously shuffle the magnetic field lines emerging from the photosphere (forcing on the velocity field, $u$). The other consists in injecting a flux of Alfvén waves at the bottom boundary, corresponding to the assumption that this waves are generated in the deeper layers of the atmosphere and transmitted through the photosphere (forcing on the outgoing Elsässer field, $\mathbf{z}^+\). A relevant difference with respect to the simplified, semi-analytical model, is that the relative phase shift between the waves determines the efficiency of the nonlinear interactions, affecting the development of the turbulent cascade. It turns out that low frequency waves, which are necessary to produce the reflected waves, are not able to drive the turbulent dynamics unless they are uncorrelated, since the interaction between a low frequency, long living wavepacket and its reflected component is weakened by the fact that they are in opposition of phase. As a consequence, low driving frequencies yield a very low turbulent dissipation for the $\mathbf{z}^+$ forcing case, since wave are free to escape the domain from below (open boundary conditions are imposed at the top but reflection is more efficient there, so a few of the injected energy leaks from the top of the chromosphere). On the contrary, when the $u$ forcing is applied, waves are trapped inside the chromosphere and have longer time to interact nonlinearly. The above mechanism slows down the nonlinear interactions, which, however, sooner or later become efficient, activating the turbulent cascade. An important limitation comes also from the boundary conditions at the top of the chromosphere, where no backward propagating waves are present. This reduces the nonlinear interactions in a region where reflection is higher and the wave amplitudes grows, underestimating the actual turbulent dissipation.
Which is the more realistic boundary condition for the photosphere is still unknown and has not a simple answer, probably none of the two extreme cases given here apply strictly. It would be interesting to use the above distinction in examining the photospheric wavenumber and frequency spectra inferred from observations, in order to understand if one of the two scenarios is verified or to what extent their combination can reproduce the photospheric complex dynamics.

The application of the model to the corona reveals clearly the above “correlation” weakening: a low frequency, time-correlated driving produces a dissipation per unit mass which is four time smaller than that obtained with uncorrelated fluctuations. In the latter case the profile of the dissipation is peaked at about $4 R_\odot$ but its value is an order of magnitude smaller than that obtained by the phenomenological model, although it seems to be proportionally bigger around $2 R_\odot$. However, the level of dissipation close to the base is affected by the boundary conditions which inhibit the outgoing mode at high wavenumber, weakening the cascade. An quick and straightforward improvement consists in joining the chromospheric and photospheric layer, through a discontinuity (as in the semi-analytical model) or specifying an ad hoc profile for the density (in this case a non-uniform grid is necessary). Downward propagating waves will come form the above corona, populating the top of the chromosphere (where reflection is very high) and outgoing propagating waves will be transmitted to the low corona, even at the small wavenumbers: the resulting turbulent dissipation should be largely enhanced with respect to the separated case studied here. In this way the problem of imposing correct boundary conditions is limited to the photospheric plane only and the heating region should shift closer to the coronal base, as demanded by solar wind models.

The amplitude of the rms fluctuations in the solar wind are in good agreement with the observational constraints. In the distant solar wind the turbulent cascade is still well developed, despite inhomogeneity of the background fields are weakened. Interactions among the outgoing and ingoing mode are limited to the interaction between the outward propagating waves and the ingoing modes produced locally by the anomalous reflection. These mode has the same phase of the mother wave and propagate in the same direction, advected by the wind, at a smaller speed. The level of cross helicity indicate a sharp dominance of the outward component but turbulence is still active, as observed in the Alfvénic part of the spectrum. However, we are not able to reproduce the evolution with distance of the frequency spectrum, as given by spacecraft measurements. In our model, no parallel spectral transfer occurs but the modifications of the frequency spectra comes from the coupling of reflection and propagation to the turbulent dissipation in the perpendicular planes. Hence, the evolution of the the frequency spectrum reflects the evolution of the perpendicular spectra, which, however, is weak. A feature in remarkable contradiction with observation is also the persisting of the signature of the
forcing in the frequency spectrum, at large distance. On purpose, it would be interesting to study the evolution of a system in which low frequency, coherent waves and uncorrelated fluctuations coexist in order to test if the resulting dissipation is enhanced by the interaction of the two populations and to what extent a pure perpendicular turbulence is able to destroy the low frequency correlation of the driving, eventually reconciling with observation. Such an experiment has already been simulated by Dmitruk et al. (2004), evolving the reduced MHD equations in an slightly inhomogeneous background medium. The authors concluded that discrete modes (a plane wave forcing) and turbulence (a broad band spectrum) coexist, and no appreciable damping of the forcing is observed. However the inhomogeneity of the equilibrium field was quite weak and moreover the simulation domain had very limited radial extent, demanding further investigations.

Concluding, turbulence is a key feature in the understanding of the heating and acceleration of the solar wind. We have prooved that both in the chromosphere and in the low corona a turbulent cascade develops and it is able to transfer energy at small spatial scales, as requested by current solar wind models, although some controversial facts still remain to be clarified. From our simple, self-consistent model of turbulent solar wind, it emerges that other (compressible) processes must be invoked to provide the necessary heating, although several limitations are imposed on the above study, the most serious being that stationary solutions are sought. To what extent the cascade proceeds in the parallel wavenumber (or frequency), as assumed in many solar wind models, cannot be investigated in the context previously described and different approaches must be followed (i.e. direct numerical simulations).

On the other hand, the evolution of turbulence seems to be well described in its incompressible, Alfvénic range at large distance. What is missing is the connection to the driver close to the Sun, in other words, the input spectrum. A step toward the resolution of the problem is hence necessarily given by the understanding or the response of the photosphere to the solicitations coming from the convection zone. This would help in isolating, in the observed photospheric spectra, the spatial and temporal scales which contain the energy that can be released for the coronal heating. Such constraints would lead to a further refined selection of the various mechanisms proposed so far as the primary drivers for coronal heating and acceleration of the solar wind.
Appendices
Appendix A

Time Scale Separation

Consider the full MHD equations for a ionized plasma made of protons and electrons, the continuity, momentum and induction equations are:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0, \tag{A.1}
\]

\[
\rho \left( \frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} \right) = -\nabla p + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B} - \frac{GM_\odot}{r^2} \rho \hat{r} + \nu \nabla^2 \mathbf{U}, \tag{A.2}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}. \tag{A.3}
\]

Now adopt a separation in slow timescales for the equilibrium fields (\(\mathbf{U}, \mathbf{B}\) and \(\rho\)) and a fast timescales for the fluctuating fields (\(\mathbf{u}, \mathbf{b}\) and \(\tilde{\rho}\)). Assume also colinearity between the magnetic and gravitational fields and that the mean flow and magnetic field are radial and depend only on the radial coordinate, as well as the density. The fluctuating fields instead are incompressible and transverse with respect to the mean radial flow.

The fields in the above equations are hence decomposed as:

\[
\rho_{\text{tot}} = \rho(r) + \tilde{\rho}(\mathbf{r}), \quad \text{with} \quad \langle \rho_{\text{tot}} \rangle = \rho, \tag{A.4}
\]

\[
\mathbf{U}_{\text{tot}} = \mathbf{U} + \mathbf{u} = U(r, t) \hat{e}_r + u(r, t) \hat{e}_\perp, \quad \text{with} \quad \langle \mathbf{U}_{\text{tot}} \rangle = \mathbf{U}, \tag{A.5}
\]

\[
\mathbf{B}_{\text{tot}} = \mathbf{B} + \mathbf{b} = B(r, t) \hat{e}_r + b(r, t) \hat{e}_\perp, \quad \text{with} \quad \langle \mathbf{B}_{\text{tot}} \rangle = \mathbf{B}. \tag{A.6}
\]

where angle brackets represent an average taken on timescales much greater than the characteristic timescales of the fluctuating fields, the parallel (||) and perpendicular (\(\perp\)) subscripts are relative to the radial direction and the fluctuations \(\mathbf{u}, \mathbf{b}\) are complex fields. Assume also that the dissipative and resistive coefficients, \(\nu\) and \(\eta\), acts only on the fast perpendicular fluctuating fields, which are considered to vary on small perpendicular length scales compared to the radial smooth mean fields, in other words we consider that \(\nabla^2_\perp \gg \nabla^2_{||}\).

To obtain the equations for the mean field, first substitute the decomposed variables in eqs. A.1-A.3, then take an average on the long time scales. Accordingly, the fluctuations equations are obtained subtracting the mean field.
equations to the decomposed version of the MHD equations above. Consider first the continuity equation, the mean and fluctuating fields equations are:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = - \langle \tilde{\rho} (\nabla \cdot \mathbf{u}) + \mathbf{u} \cdot \nabla \tilde{\rho} \rangle, \tag{A.7}
\]

\[
\frac{\partial \tilde{\rho}}{\partial t} + \nabla \cdot (\tilde{\rho} \mathbf{U}) = \langle \tilde{\rho} (\nabla \cdot \mathbf{u}) + \mathbf{u} \cdot \nabla \tilde{\rho} \rangle - \mathbf{u} \cdot \nabla (\rho + \tilde{\rho}) - (\rho + \tilde{\rho}) (\nabla \cdot \mathbf{u}), \tag{A.8}
\]

The term on the RHS of eq. A.7 must vanish if \( \rho = \rho(r) \). Then it implies that also \( \tilde{\rho} = \tilde{\rho}(r) \) so that also the RHS of eq. A.8 vanishes. Finally one is left with the same equation for both the mean and fluctuating density, without loss of generality one can than assume \( \rho_{\text{tot}} = \rho \).

The continuity, momentum and induction equation become:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0, \tag{A.9}
\]

\[
\rho \left( \frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} \right) = -\frac{GM_{\odot}}{r^2} \rho \hat{e}_r - \nabla \left( p + \frac{\langle |\mathbf{b}|^2 \rangle}{8\pi} \right) + \frac{\mathbf{b} \cdot \nabla \mathbf{b}}{4\pi} - \frac{\rho \mathbf{U} \cdot \nabla \mathbf{U}}{8\pi}, \tag{A.10}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \langle \mathbf{b} \cdot \nabla \mathbf{u} - \mathbf{u} \cdot \nabla \mathbf{b} \rangle = 0. \tag{A.11}
\]

The last equalities in eq. A.11 descend form the vector expression in spherical coordinate:

\[
(a \cdot \nabla c) = -\frac{1}{2A} \frac{dA}{dr} \Re(ac^*) \quad \text{with} \quad a = a \hat{e}_r, \ c = c \hat{e}_r. \tag{A.12}
\]

where the asterisk means conjugated fields, \( \Re \) is the real part and the area expansion \( A(r) \) is used to generalize the expression to nonspherical expansion of the magnetic flux tube \( A = r^2 \) in spherical expansion). The same relation applied to momentum equations yields the acceleration exerted on the wind by the waves whose explicit expression will be given below.

The energy equation for the mean flow is written neglecting conductivity, radiative losses and includes only a general term for the heating function, \( Q \) (having the dimension of a work per unit time) which is due to the dissipation of small scale fluctuations:

\[
\frac{\partial p}{\partial t} + \mathbf{U} \cdot \nabla p = -\gamma (\nabla \cdot \mathbf{U}) p + (\gamma - 1) Q. \tag{A.13}
\]

The momentum and induction equations for the fluctuations are instead:

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{u} - \frac{1}{4\pi \rho} \mathbf{B} \cdot \nabla \mathbf{b} = -\frac{1}{\rho} \nabla \left( p - \frac{\mathbf{b}^2}{8\pi} \right) + \frac{1}{4\pi \rho} \mathbf{b} \cdot \nabla \mathbf{B} - \mathbf{u} \cdot \nabla \mathbf{U} \tag{A.14}
\]

\[
+ \frac{1}{4\pi \rho} \mathbf{b} \cdot \nabla \mathbf{b} - \mathbf{u} \cdot \nabla \mathbf{u} - \frac{1}{4\pi \rho} \langle \mathbf{b} \cdot \nabla \mathbf{b} \rangle + \langle \mathbf{u} \cdot \nabla \mathbf{u} \rangle + \frac{1}{8\pi \rho} \nabla \langle \mathbf{b}^2 \rangle + \nu \nabla^2 \mathbf{u}
\]
\[
\frac{\partial b}{\partial t} + U \cdot \nabla b - B \cdot \nabla u = -u \cdot \nabla B + b \cdot \nabla U - (\nabla \cdot U) b \\
+ b \cdot \nabla u - u \cdot \nabla b - (b \cdot \nabla u - u \cdot \nabla b) + \eta \nabla^2 b,
\]

for which the dissipative coefficients, \( \nu, \eta \) have been considered, acting on smaller perpendicular length scales. It is now useful to introduce the Elsässer variables \( Z^\pm = u \mp \text{sign}(B) b / \sqrt{4\pi \rho} \), which represent Alfvén waves propagating outward (+) and inward (-) for a reference frame positioned on the sun. The equations describing their evolution are obtained summing and subtracting eq. A.14 and eq. A.15 divided by \( \sqrt{4\pi \rho} \). After a bit of algebra one gets:

\[
\frac{\partial Z^\pm}{\partial t} + [(U \pm V_a) \cdot \nabla] Z^\pm + (Z^\mp \cdot \nabla) (U \mp V_a) + \frac{1}{2} (Z^\mp - Z^\pm) \nabla \cdot \left( V_a \mp \frac{1}{2} U \right) \\
= \frac{1}{\rho} \nabla p_{\text{tot}} - [(Z^\mp \cdot \nabla) Z^\pm - \langle (Z^\mp \cdot \nabla) Z^\pm \rangle] + \nu^+ \nabla^2 Z^\pm + \nu^- \nabla^2 Z^\mp \quad (A.16)
\]

where \( V_a = B / \sqrt{4\pi \rho} \) is the Alfvén speed and the dissipative coefficients are conveniently written as \( \nu^\pm = 1/2(\nu \mp \eta) \). Note that in writing the time derivatives of the \( Z^\pm \) the stationarity of the density is assumed, hence the above equations are less general than eqs. A.14-A.15. In the RHS the nonlinear terms include the total (magnetic plus gas) pressure, since in the incompressible case the pressure may be written as the product between \( Z^+ \) and the gradients of \( Z^- \) and viceversa, the nonlinearities couple only counterpropagating waves. In the LHS the linear terms can be divided in “WKB” terms (advection and diagonal reflection) and “mixing” terms (off-diagonal reflection), the latter couple linearly the equations while the former are responsible for the radial dependence of the wave amplitude at high frequency (i.e. short parallel wavelength).

The energy equation for the fluctuating fields can be obtained multiplying eqs. A.16 by \( 1/2 \rho Z^\pm \) and then adding the two resulting equations. The mean field energy equation is obtained as usual combining the momentum equation multiplied by \( U \) and the energy equation. One finally gets:

\[
\frac{\partial \epsilon_w}{\partial t} + \nabla \cdot \phi_w + U \cdot f_w = -Q. \quad (A.17)
\]

\[
\frac{\partial \epsilon_T}{\partial t} + \nabla \cdot \phi_T - U \cdot f_w = Q + \rho g \cdot U \quad (A.18)
\]

where gravity is written explicitly in the RHS of the mean field energy equation. The total wave energy \( \epsilon_w \), the wave flux \( \phi_w \) and the force exerted by the
waves on the wind (and vice versa) $f_w$ are defined as:

$$
\varepsilon_w = \rho \frac{1}{4} \left( |Z^+|^2 + |Z^-|^2 \right) = \rho \left( \frac{u^2}{2} + \frac{1}{2\pi} \right)
$$

(A.19)

$$
\phi_w = \rho \frac{1}{4} \left( (U + V_a) |Z^+|^2 + (U - V_a) |Z^-|^2 + \frac{1}{2} U |Z^+ - Z^-|^2 \right)
$$

(A.20)

$$
f_w \cdot U = \rho \frac{1}{2} \left( \Re \left[ (Z^- - \nabla U) Z^+ + (Z^+ - \nabla U) Z^- \right] - \frac{1}{4} U \cdot \nabla |Z^+ - Z^-|^2 \right)
$$

(A.21)

and the total energy ($\varepsilon_T$) and the total flux ($\phi_T$) include the kinetic and internal energy ($\varepsilon$) of the mean flow:

$$
\varepsilon_T = \rho \varepsilon + \frac{1}{2} \rho U^2 = \frac{1}{\gamma - 1} p + \frac{1}{\gamma} \rho U^2
$$

(A.22)

$$
\phi_T = \rho U \left( \frac{U^2}{2} + \varepsilon + \frac{p}{\rho} \right) = \rho U \left( \frac{U^2}{2} + \frac{\gamma p}{\gamma - 1} \right)
$$

(A.23)

Coupling the two systems one finally gets the global energy balance:

$$
\frac{\partial}{\partial t} (\varepsilon_w + \varepsilon_T) + \nabla \cdot (\phi_w + \phi_T) = +\rho g \cdot U
$$

(A.24)

where gravity is the only external force acting on the system, and satisfies the relation

$$
\frac{\partial \rho \varepsilon G}{\partial t} + \nabla \cdot (\rho \varepsilon G U) = -\rho g \cdot U
$$

(A.25)

in which $\varepsilon_G$ is the potential energy associated to the gravitational field.

It is to note that the derivation given here results in a more general system of equations than that obtained in the context of RMHD (Rosenbluth et al. 1976; Strauss 1976; Zank & Matthaeus 1992), in that we allow a non uniform compressible equilibrium flow and a non uniform radial mean magnetic field, although the derivation is less rigorous. The incompressible fields, however, have the same vector form and coordinate dependence as in RMHD.
Appendix B

Shell Model Coefficients

To obtain the coefficients of the shell model, some constraints are imposed in order to satisfy the conservation of the total energy, helicity and magnetic helicity, which is proved to hold in MHD turbulence in the inertial range (i.e. neglecting forcing and dissipation). The conservation of total energy and helicity is equivalent to the conservation of the Els"asser energies, for each mode separately. Consider the variation of the Els"asser fields as given by the nonlinear interactions:

\[
(d_t Z_\pm)_{NL} = i k_n \left( a_1 Z_{n+1}^\pm Z_n^\mp + a_2 Z_{n+1}^\mp Z_n^\pm + \frac{a_3}{\lambda} Z_{n+1}^\pm Z_{n-1}^\mp + \frac{a_4}{\lambda} Z_{n+1}^\mp Z_{n-1}^\pm \\
+ \frac{a_5}{\lambda^2} Z_{n-1}^\pm Z_{n-2}^\mp + \frac{a_6}{\lambda^2} Z_{n-1}^\mp Z_{n-2}^\pm \right) \tag{B.1}
\]

Let \( \xi_n^{s_1s_2s_3} = Z_{n-1}^s Z_n^s Z_{n+1}^s \) and multiply the above eq. B.1 by the complex conjugate of \( Z_\pm^\pm \). Summing over the shell index one obtains the variation of the total energy in a plane, for each mode, \( E_\pm = \sum_n |Z_\pm^n|^2 \), due to nonlinear interactions:

\[
(d_t E_\pm)_{NL} = 2 \Re \left[ \sum_n (d_t Z_n^\pm)_{NL} (Z_n^\mp)^* \right] \tag{B.2}
\]

\[
= 2k_0 \Im \sum_n \lambda^p \left[ a_1 \xi_{n+1}^{\mp\mp} + a_2 \xi_{n+1}^{\pm\mp} + \frac{a_3}{\lambda} \xi_{n}^{\pm\pm} + \frac{a_4}{\lambda} \xi_{n}^{\mp\pm} + \frac{a_5}{\lambda^2} \xi_{n-1}^{\pm\mp} + \frac{a_6}{\lambda^2} \xi_{n-1}^{\mp\pm} \right].
\]

Since \( Z_\pm^\pm(t) \) and \( Z_\pm^\mp(t) \) are independent, \( \xi_n^{s_1s_2s_3} \) is independent of the index \( n \) and one can shift the indexes in the summation so that they are equal to \( n \). Conservation of the total energy results in:

\[
a_4 = -a_1, \quad a_5 = -a_3, \quad a_6 = -a_2 \tag{B.3}
\]

The third invariant in 2D is the ananstrophy, \( H_B^{2D} = |A|^2 \) with \( B = \nabla \times A \), which reduces to \( H_B^{(\alpha)} = \sum_n |b_n|^2 / k_n^\alpha \), with \( \alpha = 2 \), in the shell model (Frick & Sokoloff

\[1\] with the condition \( Z_m = 0 \) for \( m < 0 \) and \( m > n_{\text{shell}} \)
1998). Its variation, due to nonlinear interaction is given by:

\[
(d_t H_B^{\alpha})_{NL} = 2k_0 \Im \sum_n \left( \lambda^{n(1-\alpha)} A_1 ([bub]_{n+1} - [bub]_{n+1}) + \frac{A_2}{\lambda} ([ubb]_n - [bub]_n) \\
+ \frac{A_3}{\lambda^2} ([bub]_{n-1} - [ubb]_{n-1}) \right)
\]

(B.4)

Here \( A_1 = a_1 - a_2, \ A_2 = a_1 + a_3, \ A_3 = a_3 - a_2 \), and \([pqr]_n = p_{n-1} q_n r_{n+1} \). Again, aligning the indexes in the summation one obtains:

\[
\lambda^\alpha A_1 = A_2, \quad \lambda^{2\alpha} A_1 = A_3
\]

(B.5)

A particular class of shell models is selected imposing \( a_1 + a_2 = 1 \), in this way for \( b_n = 0, \ d_n u_n \) reduces to the hydrodynamical GOY shell model\(^2\). With this normalization one gets:

\[
A_1 = -\frac{\lambda^{-2\alpha}}{1 - \lambda^{-1}}, \quad A_2 = -\frac{\lambda^{-\alpha}}{1 - \lambda^{-1}}, \quad A_3 = -\frac{1}{1 - \lambda^{-1}}.
\]

(B.6)

or equivalently the shell model coefficients are

\[
a_1 = \frac{\delta + \delta_m}{2}, \quad a_2 = \frac{2 - \delta - \delta_m}{2}, \quad a_3 = \frac{\delta_m - \delta}{2},
\]

\[
a_4 = -\frac{\delta + \delta_m}{2}, \quad a_5 = -\frac{\delta_m - \delta}{2}, \quad a_6 = -\frac{2 - \delta - \delta_m}{2},
\]

(B.7)

with \( \delta \equiv 1 + \lambda^{-\alpha}, \ \delta_m \equiv -\lambda^{-\alpha}/(1 - \lambda^{-\alpha}) \). Setting \( \lambda = 2 \), a common choice, \( \delta = 5/4, \ \delta_m = -1/3 \).

**Scale by Scale Energy Budget Equation**  The scale by scale energy budget equation is obtained by multiplying eq. (4.1) by the complex conjugate \( Z_n^{\pm} \) and summing over the smallest wavenumber. Assuming equal resistivity and viscosity and explicitly adding the forcing terms \( (f_n) \), one gets:

\[
\frac{\partial E_N^\pm}{\partial t} + (U \pm V_o) \frac{\partial E_N^\pm}{\partial r} + 2R_{WK}\Sigma_N^\pm + R_{MIX}^\pm \Sigma_N + \Pi_N^\pm = -\Omega_N^\pm + \mathcal{F}_N^\pm
\]

(B.8)

where the following quantities, the cumulative energy at scale, \( l \leq 1/k_N \ E_N^\pm \), the cumulative dissipation, \( \Omega_N^\pm \), the cumulative energy difference, \( \Sigma_N \), the cumulative forcing, \( \mathcal{F}_N \), are defined as:

\[
E_N^\pm = \sum_{n<N} |Z_n^\pm|^2, \quad \Omega_N^\pm = \sum_{n<N} \nu k_n^2 |Z_n^\pm|^2
\]

\[
\Sigma_N = \sum_{n<N} (|u_n|^2 - |b_n|^2), \quad \mathcal{F}_N = \sum_{n<N} f_n
\]

(B.9)\( ^2\)Imposing such a relation for two coefficients results in a normalization of the coefficients of the nonlinear terms.
and the energy flux $\Pi_N$, from scale $l_N$ down to scales $l < l_N$, is given by eq. B.2 with the opposite sign and the sum limited to $n < N$:

$$\Pi_N = -2k_N S \left[ a_1 Z_N^+ Z_{N+1}^+ Z_{N+2}^- + \frac{a_2}{A} Z_N^+ Z_{N+1}^- Z_{N+1}^- + \frac{a_3}{A} Z_N^- Z_{N+1}^- Z_{N+1}^- \right]$$  \hspace{1cm} (B.11)

The reflection coefficients are grouped in a diagonal term (WKB) and an off-diagonal term (MIX), with reference to eq. 4.2-4.3, their expression is given by:

$$R_{WKB}^\pm = \mp R_{iso}^\pm = \frac{1}{2} \left( \frac{1}{A} \frac{dA}{dr} + \frac{1}{V_a} \frac{dV_a}{dr} \right) (U \mp V_a),$$  \hspace{1cm} (B.12)

$$R_{MIX}^\pm = R_{pol}^\pm \pm R_{iso}^\pm = -\frac{1}{2} \frac{1}{V_a} \frac{dV_a}{dr} (U \mp V_a).$$  \hspace{1cm} (B.13)

The former is responsible for the the growth of the fields when a high frequency (small parallel wavenumber) wave propagates in a non-uniform medium (WKB variation), the latter is responsible for the generation of a species from the other and is zero only when the magnetic and kinetic fields are in equipartition, or equivalently one of the two species vanishes (referred as mixing terms since they couple one species to the other).

On the LHS of eq. B.8 the 3 terms after the time derivative all depend on the variation of the fields along the radial direction and are responsible for the linear reflection and propagation effects. The WKB term depends on the inhomogeneity of background field while the mixing term depends also on the frequency of the fluctuations (or equivalently their parallel wavenumber). In their absence (or assuming that they are negligible) the energy balance equation tells us that the rate of change of the energy at scales equal and bigger than $l \approx 1/k_N$ is given by the energy dissipated ($\Omega_N$) and the energy injected by the forcing ($F_N$) at such scales minus the flux of energy to smaller scales ($\Pi_N$) due to nonlinear interactions. Assuming a statistical stationary state, that dissipation acts only at small scales ($l_\nu$) and that the forcing at the contrary is applied at large scale ($L$), one gets $\Pi_l = F$ in the inertial range ($l_\nu \ll l \ll L$). In fully developed turbulence it is assumed that in the limit of vanishing dissipative coefficients dissipation attains a finite (non zero) value, $\epsilon > 0$. Taking this limit one finally gets $\Pi_l = \epsilon$, that is in the inertial range the energy flux is independent of the scale considered (i.e. uniform in the wavenumber).
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